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HISTORY, THEORY, AND TECHNIQUE

OF

STATISTICS.

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PART II.

THEORY AND TECHNIQUE OF STATISTICS.

INTRODUCTION.

§ 59. POSITION OF STATISTICS AS A SCIENCE.

THAT statistics has long since attained the importance and general recognition of a science is shown by the history of its development. This history also demonstrates the complete unanimity which exists in regard to the purpose and processes of the statistical method. None the less the scientific character of statistics, and its position in the general system of sciences, have remained a matter of controversy.

A conflict of opinion still exists on the question, whether statistics is to be considered as a science of things or a science of method.

One group of statisticians considers statistics as a science to be wholly independent of any distinct method, but as attached to certain objects, constituting our scientific knowledge of them. These investigators either hold fast to the opinion that the knowledge of the State, conceived somewhat as Achenwall formulated it, is the only true and valuable object of statistics as a science, or, with Süssmilch, they consider the reign of law in the phenomena of human existence, in the physiological and moral occurrences of the life of nations, as the peculiar content of statistical inquiry.

Another group recognizes the importance of the statis-

tical method as such, and finds in its application to the conditions of nations and peoples or human communities the field of scientific statistical knowledge.

A third party finally declares statistics to be exclusively a science of method, applicable to every object—to be applied logic, and to be classed with other independent methodological sciences, such as logic, mathematics, critique, and hermeneutics.

It must be clear that the questions, whether statistics is a science, and if so, what is its place in the general system of sciences, cannot be decided upon the basis of statistics itself. To bring into clear relief the many related ideas and manifold actual connections of thought, and assign to every field of investigation its proper place, would require a comprehensive analysis of all human knowledge, a general theory of science.

The lack of such a standard for the sciences, which on account of the necessity of metaphysical premises, must, in any case, receive a coloring from the individual temperament, makes itself very keenly felt in the development of the system of statistics. A clear *definition* of the sphere of ideas to be considered is demanded not only for the elegance of the treatment, but as an almost indispensable proof of a clear and precise system of ideas.

Statistics is therefore called upon all the more urgently to form its own ideas into a sound theoretical system, taking into account all particulars and details.

§ 60. POINTS OF VIEW FOR THE THEORY OF STATISTICS.

Obviously the theory of statistics cannot treat of the actual situation of innumerable phenomena, which have so long been ascertained in civilized countries, and which vary as they do from year to year. How many inhabitants there may have been in Rome at a certain period, how great the public debt of Spain, how extensive the railroad traffic in Russia, etc., are facts of interest merely for the

description or history of Rome, Spain, and Russia. And if on the average 21 boys are born for every 20 girls, it is not for statistics but for anthropology to examine whether this statistical probability can or must be accepted as final. Even in the most exclusive circle of Achenwall's school, the theory of statistics treats of the system and not of the contents of the "Staatsmerkwürdigkeiten." The problems which practical and theoretical statisticians are called upon to solve are part of the theory of statistics, not with reference to their anthropological, geographical, political, or economic importance, but as the subjects of different varieties of investigation. Each case is for the theory an illustration or a problem.

The theoretical point of view must necessarily be concerned with the propriety and practicability of the plan of observation, the manner of execution, the reliability and utility of the results, the combination of the facts, and the correctness of the conclusions. These are all questions of method. For all these phases the theory needs authoritative principles, an understanding of the relations, and the means for facilitating an examination and judgment of them.

The theory of statistics must therefore be based on the statistical method. While it penetrates to the foundation of this peculiar procedure, tested by long experience, the logical contents of the method must on the one side develop into a system the theory of statistics, and upon the other it must appear what form the technique of statistics should assume in order to satisfy the requirements made by the statistical method, theoretically and critically developed.

What in general should be regarded as statistical method, which is the subject of our investigation, can be gathered from its relation to logic, and from a survey of its specific leading thoughts.

§ 61. RELATION OF THE STATISTICAL METHOD TO LOGIC.

From the history of statistics, the method appears to us *empirically* as a procedure of obtaining judgments and conclusions as to the relations of a mass of changeable and variable things, by an enumeration of characteristic qualities.

More briefly the same thought may be expressed as the method of judging collective phenomena from the results of enumerations.

Scientific logic might, as §§ 57 and 58 show, express the contents of this sentence more precisely, thus: the statistical method is a process, based on an enumeration of characteristic phenomena, of forming empirical judgments and conclusions relating to the varied and complicated aggregates of existence. It is used where the experimental investigation of an individual or collective object is possible neither by induction, the conclusion from the special to the general, or by deduction, the conclusion from the general to the special.

The meaning of the definition must be explained. All actual existence appears to man as inexhaustible multiplicity. It appears to his thinking powers as made up of certain things with constant qualities despite the changing appearances. From the constancy of these qualities we form ideas.

Of the things thus divided by ideas we know actually but few exactly, by closer view, or strict experiment; to the rest, on finding single qualities, we apply, inductively or deductively, the presumption of similar qualities, and, as experience shows, the nature of things permits this in a degree sufficient for ordinary purposes.

No one thing is however actually identical with another, no one can occupy the space of another, and all things which follow each other in time are different. Each is an individual, and has, as such, certain qualities which dis-

tinguish it from every other. All things can be considered, according to the point of view, as single, or as in combination, forming larger things. Here we derive the ideas of individuals and collectives or aggregates. Each one of them can be dissolved finally into a multiplicity of things simply inexhaustible for human observation, and as difficult of complete comprehension in its constitution as in its variations. Each thing forms, therefore, according to the conception of the moment, a unit or an aggregate. Considered as an aggregate, it is divisible even to the atoms, into the most various constituent objects. The investigation of all these parts, which appear, according to the point of view, as individuals or collectives, exceeds the power and resources of any observer. According to its nature, such an accumulation of dissimilar objects permits no satisfactory knowledge of its full content, of the inner relations, and of the possible expectation of permanency or change, by induction or deduction, which could only deal with the single objects. Much less is the possibility conceivable of acquiring by experiment any knowledge of the continually changing multiplicity in each single thing, and in mutual relations and conditions of them all.

The method of statistics, immaterial whether it be considered as a general one of all empirical science, or as specifically statistical in every case of its application, appears therefore as the means of obtaining knowledge as to aggregates, *i. e.*, things conceived in the confused and variable multiplicity of their components.

§ 62. THE LEADING THOUGHTS OF THE STATISTICAL METHOD.

The method of statistics must unconditionally abandon the effort to investigate completely the inexhaustible different combinations of the aggregates. But its *fundamental* idea is to penetrate in a limited, yet systematic way, into the aggregate in order to ascertain whether and how often certain things are to be found there, or how often they

pass in and out of it. This search and enumeration is the simplest observation possible, not influencing in any way the condition of the aggregate. The result can, in the first instance, be only a more or less defective description. The statistical method aspires, however, to draw conclusions from such enumerations, as to the relative measure of the phenomena, the causal connection, and the expectation of regular reappearance of the things counted.

The idea is, thus, to approach the investigation of the multiplicity, with a certain *purpose* in view, and to observe merely such things whose number in the particular aggregate it is necessary to ascertain for this purpose.

Things which are to be sought must be known beforehand. When it is purposed to find their number the idea chosen in advance must designate something capable of enumeration which can serve as the unit of the count. This presupposes that it can be distinguished in the confused and changing real multiplicity of the aggregate as a certain object, with definite perceptible qualities.

All objects whose characteristics correspond with those of the unit selected are to be counted. With the sum which results from this counting the first step demanded by the statistical method, the *methodical enumeration*, has been taken. As far as the purpose of the problem requires the enumeration of a group of different things instead of that of a definite thing, it must take place in the same way for each distinct thing.

The description, by means of the number of any kind of things, can form the basis of a methodical judgment only when the number can be measured. The meaning is not at once apparent, but depends upon the relation to a certain scale. This scale can only be derived by seeing how large or small is the number of units in the aggregate investigated in comparison with the average number of the same units found in analogous aggregates. Such aggregates are analogous which, considered either as individuals or collectives, come in the same category. The

comparison may be as different, according to the special purpose, as the conception of the aggregate as a unit. Upon this quantitative relation further judgments may be based.

Certain points for the determination of causal connections can be obtained, if the selection of the things counted had this end in view, or suffices for the purpose. The greater measure of cause must bring about the greater measure of effect. In the aggregate investigated only that measure of things can be recognized as an essential or prominent cause which, by comparison with analogous aggregates, appears to be in a functional relation to the effect. When units which express the effect or cause directly are not suited to enumeration, others must be selected which furnish an indirect or symptomatic evidence. If no functional relation exists between the units which have been chosen each time with reference to probable causal relations, we have simply discovered that of the possible causes considered all are inapplicable.

From the judgment as to causality the last step leads to that of probability. The same causes bring about the same effects under the same conditions. Under like circumstances the average measure of things in analogous aggregates would also be the most probable in that to be investigated. The average relation of cause and effect which appears among them could also be considered as the regular one. When regularities are observed they are to be considered as based upon like circumstances. With this judgment as to the expectation of phenomena the last aim of the problem of penetrating into the multiplicity of the aggregate is attained.

On account of the necessity of observing and counting considerable numbers of distinct units in a series of analogous aggregates this process requires generally very considerable expenditure of effort. Nevertheless, it leads only to a partial description, and at the most to an approximate, an hypothetical judgment as to the inner structure of the

investigated aggregate. But the impossibility of investigating each single thing with experimental precision is common to all empirical sciences. Inductive and deductive judgments are also hypothetical. The results of the statistical researches do not refer so much to the typical phenomena, which are the premises for inductive and deductive reasoning, but consist rather in a *classification of the non-typical*, which is unattainable in any other manner.

Formally, the procedure consists of two stages, one preparatory, which counts the units in the aggregate selected for the specific purpose, and describes the same by means of totals; the other, or completing stage, which, obtaining for comparison the sums from enumerations of similar units in different analogous aggregates, forms judgments on the relations of quantity, of the causality and the probability or regularity of the phenomena. The technique of the method follows closely these logical steps.

The theory of statistics has for its object to establish the proper foundation for each stage of the process, and thus determine the principles according to which in a given case we may test the accuracy of statistical results and the mode of obtaining them.

A. THE PRINCIPLES OF ENUMERATION.

I. FUNDAMENTAL CONCEPTIONS.

§ 63. EMPIRICAL CONDITIONS OF THE ENUMERATION.

The first step of the statistical method indicated by the general outline of the procedure is that of the enumeration. Characteristic things in the multiplicity to be investigated are to be ascertained with respect to their number as they exist in it, or as they enter and leave it.

To define in sharp and concise terms the *conditions* of this first step as well as of the further processes is not without difficulty. In particular it may be observed that in regard to necessary requirements of the method complicated problems are often more comprehensible than the most simple ones.

It might readily appear that the enumeration of anything, whether for statistical purposes or for the purposes of daily life, a mere counting-off according to the numerical series, was a complete operation, whose aim was attained absolutely without any reference to the considerations mentioned in the preceding paragraph. That this view is erroneous appears readily by carefully observing any empirical attempt to count.

Suppose someone were asked to count paper or fruit lying before him. He would see at once that before beginning he must settle which of the objects should be considered as fruit or paper, for the purpose in view. The explanation given must be perfectly clear and perceptible to the person counting. Further, it must be settled what is a *piece* of paper, or fruit, and whether, perhaps, fruit is to be counted by piece or weight, or the paper by the size of the sheets. A change in these particulars could not take place during the count. Besides, it must be clearly defined what the words "*before him*" express; that is, there must be evidently a *limit* of space. There must be a limit of time during which pieces should not be removed nor added. If objects change their positions during the count, they must be considered as changing objects, and measures must be framed for their treatment. Simple as these conditions may appear, *no one can remain unfulfilled* if the result of the enumeration is to be correct or in accordance with the intentions of the questioner.

They need not, it is true, always be expressed, but the most complete understanding must exist between the questioner and the counter as to the smallest detail. It is not to be doubted that these requirements are

alike indispensable for the greatest and smallest problems in enumeration. What, however, is their theoretical significance?

§ 64. ENUMERATION AND CALCULATION.

The complex conditions of the empirical process of counting indicate that these peculiarities have a basis in the *fundamental relations of logical knowledge*. Enumeration presupposes the idea of number. Number is one of the earliest abstractions that arose in the formation of ideas. As soon as thought grasps objects and distinguishes between them in classification, it is impossible to escape the notion of *plurality*. Things agreeing in their conceptions appear no longer as isolated things, but as several of the same species. The necessity of dealing in thought with indefinite pluralities, and of expressing them in communication to others, makes itself felt very early in the formation of language. There is hardly a language, however crude, which lacks the plural denominations. But the plural, and evidently also the dual, are not numerical expressions. The dual seems to be the expression for the frequent symmetrical double phenomena of nature, as the eyes, hands, etc. (W. v. Humboldt, *Ueber den Dualismus*, 1828.)

Number is developed first in the progress of conscious observation from one thing to another similar one. It presupposes that the idea of the thing is conceived of as a unit. Number is not formed by observing 1 and 1 and 1, or placing them in succession. It is formed when in a continuous observation each occurrence of the unit is grouped with those which have already taken place, and an expression given to each successive group. Each expression indicates the total of all occurrences of the unit up to a given point of time. Numerical terms belong to the oldest traditions in all languages, and yet the difficulty of counting and retaining numerical ideas is shown by the

fact that many races have no numerals beyond three or five.

The expressions corresponding to the successive groups of units form the numerical series. In counting concrete things memory applies the numerical order to the observations of experience to determine how often the thing which forms the unit has been found. This finding things by observation is a real operation, and where the enumeration takes place in the recollection, it is an image of the real world.

The succession of expressions in the numerical series is a *succession of totals*. The numerical series can be completely separated from concrete things and be continued without aid from them. It forms in that case a succession of ideas, each of which exceeds the sum preceding by one unit. This order permits a progress by sums more than unity, as well as a division into series of pluralities. This facilitates the use of *numerical signs*, mere figurative images of the numerical expressions.

The consideration of the abstract numbers and the relations of numerical series awakened much interest at an early date, and led to deductive conclusions from the ideas of the totals. Thus were founded calculation and mathematics, which appear to be the oldest human sciences.

It would, however, be a mistake to see in the early and astounding development of mathematical calculation also a development of enumeration. On the contrary, this art remained all the more neglected. Closer consideration shows that calculation has nothing in common with enumeration except the number. It can proceed apparently from actual existing pluralities, but only apparently, for it depends entirely upon the idea of the identity of the units. It can apply this idea to any object, but in reality the objects are not identical, and this cannot influence the idea of the units. Calculation necessarily deprives the unit with which it operates of every specific content, and perceives in each number only a certain quantity of wholly empty and abstract

units, which permit only repetitions and divisions just as empty and abstract. All judgments and conclusions of calculation are therefore analytical. Every logical thinker obtains by correct conclusions from the first premises the final results of all arithmetic without actual experience. Upon this analysis is founded the science of mathematics. With empty numerical conceptions it can combine only similar ideas of corporeal dimensions, point, line, surface, and volume. All these ideas are so empty and abstract, that they can be represented by no phenomena of actual existence, nor do they correspond to any. All constructions are entirely ideal. None of their propositions contains a synthesis. All these propositions and formulas simply relieve the investigator by the aid of memory from undertaking the almost insurmountable labor of repeating in his own thought the abstractions of centuries.

Here we see the difference between enumeration and calculation. Calculation elaborates by analysis the necessary consequences from given ideas of magnitude, without asking whether or not these ideas correspond to any reality. Enumeration endeavors to ascertain how often a designated unit is present in a limited reality and is a thoroughly real and synthetic operation. As reality is concrete, so must the unit be also, and must be found by concrete methods. From calculation, which can never undertake a large or small task in practical enumeration, the latter can obtain assistance only when the actual unit is found in typical division or multiples, so that the divisors and multiples can be calculated from the sums counted; *i. e.*, can be derived by analysis.

§ 65. ENUMERATION OF THINGS AND MEASUREMENT OF QUALITIES.

The distinctive theoretical peculiarities of enumeration require that the collective phenomena which statistics are to investigate must have the character of real independent

things in order to be counted. They may indeed exist merely in thought or be counted in memory, having their limits fixed in our imagination, but in one way or the other they must be fixed and accessible to observation. This requirement is perfectly comprehensible for well-rounded concrete objects, as men, houses, ships. It becomes a question, however, how the numerous distinctions are to be treated, which result from the qualities of such objects, and again how a phenomena may be utilized for the investigation when it does not appear to be so definitely limited.

A necessary premise of enumeration is that all the qualities of an object which characterize it as the unit of the count must be fixed and invariable for the period of the observation upon which the count is to be based. As the statistical observation is always instantaneous, or at least must be regarded so, and as the quality always attaches to an object and never exists without it, the quality cannot be variable for the statistical observation; for when the quality changes so does the object also. The change in the quality would necessitate two enumerations. In the two the object is different, as explained.

Qualities are never counted, but simply the objects possessing one quality or another. On the contrary, all *qualities* of things can be measured, and this is always necessary if it is desired to observe the exact differences in the same quality.

Every quality of a thing is perceptible to us through impressions of the senses, and upon the greater or less force of these impressions depends their measure, or their proportion. To observe the difference in which a quality appears, we seek certain easily-found limiting points, and divide the space between them into equal parts to form a scale. One division forms the unit of measure, which in an observation will be found either in multiples or fractions.

The qualities of things which are most frequently the

subject of measurement, are magnitude (measures of length, surface, volume), gravity (measures of weight), duration (measures of time), value (money, the measures of currency). Measures can also be obtained for the intensity of other qualities. For color, heat, sound, hardness, such measures are quite familiar.

Every regular progressive gradation contains the elements of a measure in the succession of its parts. But it can be utilized only when an invariable known scale is taken as a basis. The ordinances of weights and measures of the governments rest upon such bases. They should always be founded upon a normal measure, certified, as far as possible, by physics.

As in every measurement the number of the units of measure must be ascertained, it is evident that, strictly speaking, no measurement can be obtained without enumeration. Yet, counting off the degrees of a quality is different in its nature from the statistical enumeration. Statistics use it only exceptionally, partly as a supplement to the observations, partly as a means of making them objective.

The first case occurs when things of the same general class are to be distinguished in the enumeration according to *certain degrees of a quality*—e. g., horses according to ages, houses according to the number of stories. The enumeration of the degree of quality is only preparatory to the enumeration of the things differentiated. If persons are to be numbered in the census according to ages, this preparation of the classes, etc., like many other measurements not to be carried out at the instant, cannot be undertaken by the enumerating official. Distinctions easily recognized or estimated, can naturally be made in connection with his observations.

The other case occurs when the things have in themselves no *sufficient limitation* as independent objects, such as air, rain, area, arable land—time and space generally; or, indeed, actual things, such as products, grain, iron,

wood, where the number of pieces furnishes no practical nor sufficiently distinct idea. In this case the quality, measure, or weight must replace the limitation lacking in the thing itself. But the enumeration cannot, nevertheless, be performed according to the quality, but only according to the units of quality, thus established in the thing to be counted.

The quality alone can never replace the thing. One cannot count ages without the persons, or weights without the goods to which they belonged; and it is impossible to count prices, market prices, and similar data of measure or value without the articles for which they are counted or calculated. Every such general datum requires, if it is to be correct and not merely an inexact estimate, classification of the objects according to the prices paid, noting the number or the weight of the things of each class, and calculation of the price, as the average of the number of units reduced to a uniform measure. The fundamental premise of statistical enumeration, whether measurements of quality come in question or not, is, first of all, the real objective nature of the unit of enumeration.

§ 66. THE UNIT OF ENUMERATION.

Intimately connected with the requirement of the method that only concrete things can be objects of the enumeration, is the further one that these objects can only be enumerated according to a previously well-defined idea. The things to be included in the enumeration must correspond entirely with the preconceived notion of the unit of enumeration. Nothing contained in the aggregate which corresponds to this idea can be passed by unnoticed. This is the indispensable condition of the correct enumeration, and, therefore, of paramount importance as the basis of the entire process.

All essential characteristics of the unit of enumeration must be definitely fixed before the process begins. They cannot be changed by any modification or seeming expla-

nation during the process. The unit should be so defined as to leave no room for doubt. This is essential to the uniformity of the enumeration, and indispensable for the subsequent use of the results. But it is seldom possible to determine expressly for the enumerator the whole group of characteristics essential to the idea of the unit of enumeration. The common usage of the language must, with the greatest possible simplification, serve as the basis of the idea. Often it is sufficient in itself, as "man," "animal," "tree." Certain ideas have a legal meaning or one equally capable of proof, such as "letter," "merchant," "criminal." But it must always be considered whether the word in common usage is applicable to the enumeration of independent things, or requires some more precise definition, as "house," "dress," "book."

As a rule, it is necessary, besides the designation of common usage, to determine expressly certain particular characteristic qualities. This is especially true where things of the same general class are to be counted differently, according to certain features. Here the process of measuring qualities, described in § 65, comes into play. At the same time it may be observed that the ideas of quality, which seem to have great, even mathematical, exactness, do not apply strictly in reality, as, for instance, the term "spherical." Further, it must be remarked that, especially in the designation of qualities, the use of language is exceedingly variable, and can seldom dispense with an exact scale.

A systematic selection of essential characteristics facilitates the proper comprehension of them. The value of the determination of the idea agreed upon depends on the capacity of the observer. For a group of a few experienced enumerators, the units of enumeration can be defined very differently than when numerous different and inexperienced persons are drawn into the service.

§ 67. THE FIELD OF THE ENUMERATION—THE LIMITS OF TIME AND SPACE.

The consideration of a mere off-hand counting of things (§ 63), shows that, besides a definite unit, the enumeration requires a limit of the time and space in which it was to be made.

The objects which are to be counted may form either a *stationary* number, *i. e.*, be found in the same space at the same moment; or a succession or *movement*, *i. e.*, pass through the same space in a certain period of time, or they can appear variously in the given limits of time and space. If the same object be found more than once in the given time, it becomes necessary to determine whether it is to be counted but once or oftener.

For all these cases, it is to be remarked that while the objects to be enumerated may completely fill out the given limits of time and space, this is only accidental, and, according to the nature of the enumeration, rather improbable. For the problem is, to find these units in a given time and space. They may or may not be found. The needs of the enumeration have been complied with, even when the result is zero.

Thus it is evident that the limitation of time and space does not take place in the unit, but upon some connected fundament, considered as a whole, which is the field of the investigation. Such a field, limited in time and space, in which real objects are to be enumerated, must consist necessarily of a more or less extended portion of actual space, either at a given instant or for a longer period. Actual space is not a vacuum, but filled with an immense multiplicity of real things, and in the course of any longer or shorter period these can change in inconceivably numerous ways.

Each field thus limited must of necessity be conceived as a whole, as individual or collective. But every unit can, as we have seen, be considered in regard to its inter-

nal complexity, as an aggregate, and thus become the field of an enumeration.

Thus it appears that the idea of the aggregate, the investigation of which by enumeration forms the purpose of statistics, is included in the idea of an enumeration strictly considered.

II. THE PROCESS OF ENUMERATION.

§ 68. OBSERVATION AND SUMMATION.

The enumeration investigates how often the unit chosen is to be found in the given aggregate. It consists, therefore, of an observation of the units, and a summation of the observed cases of its existence.

The observation must distinguish the components of the aggregate into such things as possess the characteristics of the unit and such as do not. It must be so planned that everything which might possibly correspond to the unit shall be noticed and examined with reference to the decisive characteristics. No unit should be omitted or counted more than once. When there are several observers this must be avoided by a careful division of the field among them.

The observation is direct, when the observer searches the entire aggregate himself for them; indirect when he has his resort to records and registrations and announcements and lists of various kinds. If the fact of record is conditioned on the same qualities which make the thing a unit of enumeration, the observation is properly in the hands of those who make the records.

For records which serve statistical purposes exclusively, specific schedules are in use. They seek to obtain the notes of the single observations, systematically arranged according to the required points, with as little writing as possible, yet definite in form and convenient for summarizing. If they form a system of coördinates, in the lines

of which the distinctions of the unit of enumeration are to be arranged according to like ideas, they are called *lists*. If they are limited to a single unit, whose characteristics are so reported that afterward distinctions can be made in the summation, they are called cards. (*Statistik d. Deutschen Reiches*, Bd. I., pp. 77, 103.)

The summation is direct when the totals are given in the course of observation; indirect, when, as is usual, the single observations are noted and afterward added together. A tabular scheme, which presents the summations of the results systematically and in conformity with the purpose of a statistical problem, is called a combination formula. (Cf. pp. 99, 395 of work above quoted.)

§ 69. SUBSTITUTES FOR ENUMERATION, CALCULATIONS, ESTIMATES, AND THE ENQUÊTE.

Enumeration by observation and summation of the units is, as a rule, a very extensive, tedious, and expensive operation, as most statistical inquiries are concerned with the affairs of peoples and nations. In every case the endeavor must be to make the solution as simple as possible. But it is often altogether impossible to attain the practical end in view, unless it can be done with small means, or in a limited period of time.

Hence operations have been devised which can be used in a measure as substitutes for enumeration. Such substitutes are, however, always imperfect. They are based on the effort to use results already known, in order to dispense with the necessity of new investigations.

The simplest is the so-called "estimate." It is, when based upon a cursory observation, simply a very inexact enumeration; when it is formed without an observation it necessarily supposes previous observations on the basis of which a species of inexact enumeration is carried out mentally. To do this, however, one must be permitted to

suppose a certain degree of constancy of the units and the aggregates.

Another means to this end is sought in calculation. Enumeration seeks to determine the unknown frequency of the unit in the aggregate, but a calculation must be based upon known elements. If a unit of enumeration is a distinct function of another, the number of the latter can be calculated from that of the former; and conversely, when one such unit has been obtained by an enumeration, there is obviously no need of one for the other. But calculation cannot replace enumeration, for calculation is always analytical, never synthetical.

The use of probabilities, so frequently applied in place of enumeration, can only replace it up to a certain point. This is not because it is hypothetical. On the contrary, such a hypothetical conclusion as to a number must, where it is critically permissible, be regarded as the highest aim of statistical inquiry. It is based on the most difficult combination of the results of the various enumerations.

The method known as expert estimate or enquête is generally a combination of these various modes of obviating a direct enumeration. In the stead of actual results of enumeration, it deals with the data furnished by persons whose knowledge and capacity for judging existing observations, estimates, enumerations, elements of calculation and probability, renders them worthy of confidence.

The enquête is usually applied as a simplification and acceleration of statistical inquiries. It is for many things the only form of investigation which can be used. It cannot be dispensed with, where the organization of competent enumerators to go over the entire aggregate is not feasible; or where merely such enumerators can be obtained whose self-interest, opposition to the matter, or false notions stand in the way of correct enumeration.

But if the results of an enquête are to be statistically useful, they must be capable of replacing completely the

results of actual enumeration; they must retain the complete precision of the idea of the unit of enumeration; they must comprehend the entire aggregate; and they must express numerically and distinctly the totals estimated as if they had been counted, or at least must make it possible to calculate them.

Enquêtes which are carried on with an indefinite idea as to the aggregate and the unit, or express their opinions without any definite gradations in number and measure, are nothing more than general descriptions. They are of no use for the investigation of aggregates, and can be utilized only for such purposes as are served by a subjective description, without an exact comparison of relations.

Allied to the enquête is the so-called "political arithmetic," where it endeavors to give information of certain definite conditions. The statistician here appears as an expert. But political arithmetic concerns itself, on the one hand, with general problems in the most varied fields of statistical research, and, upon the other hand, with questions of methods and arithmetical conclusions from the results. Hence the term is a broad one.

§ 70. POSSIBILITY OF ERROR IN ENUMERATION.

Enumeration is a practical process upon actual foundations, and must therefore struggle with mistakes and imperfections. Errors in observation or in summing up are always to be apprehended. The easier it is to mistake the characteristics of the unit of enumeration, or the limits of time and space, or to judge them falsely, the more must omissions and duplications be expected. The more figures are written, and the less the calculations are proved, the greater is the probability of errors in transcribing.

The improvement of the technique must seek to limit as much as possible the sources of error, but they are no reproach to the method as such. Errors in the observation may be subsequently corrected, when the existence or non-

existence of the units falsely counted can be ascertained for the time of the original enumeration. A certain possibility of error must always remain, and the attempt must be made to estimate its extent.

For certain purposes the presumption of even small errors can render the result useless, whereas in other cases even very considerable errors may be of no consequence. The latter is particularly the case when the problem is sufficiently solved by a certain maximum or minimum number of units. The more restricted the purpose the greater may be the possibility of error, generally speaking, but the more general the purpose, the more the results must serve for indefinite, not yet known problems, the more must the limits of possibility of error be contracted. An unlimited possibility of error would render the result worthless for all purposes. A judgment with reference to this matter is indispensable to anyone proposing to use the numerical results obtained.

Properly prepared statistical enumerations must therefore include in their presentation sufficient explanation of their basis and processes to permit a conclusion as to the possibility of error, and in general to furnish the guaranty, that for the usual sphere of statistical problems, the reliability of the figures is sufficient.

§ 71. PECULIARITIES OF THE RETURNS.

The number of units present in the aggregate can be obtained directly by observation and summation, or by complicated calculations. In every case it represents a total—that is, a definite number which tells how many times the unit has been found in the mass.

This total, because a total, gives no picture of the arrangement of the units in the aggregate. It would be an egregious error to suppose that anything was known as to the position of the units, as, for instance, a map shows the positions of the cities of a country. This is by no

means the case. Every single unit is observed at the time of the enumeration, but it is not located. It disappears completely in the final result, which is the total sum of the enumerated units. The results of the observation are not susceptible, after the enumeration is complete, of any further subdivision of either time or space.

If it is possible to say, as is often done, how many of the enumerated units were found in this or that portion of the space or time in question, it must be clear that separate enumerations have been made for these portions. The same unit of enumeration has been employed, but it has been applied to an aggregate corresponding to these portions, and necessarily limited in time and space in the same manner. The aggregate to be investigated has been separated into independent minor aggregates. The results of these enumerations for the smaller aggregates may be combined to form the total for the general aggregate. But in so far as the enumeration has been divided into particular enumerations for particular aggregates, the result for each aggregate is a single total, incapable of further subdivision.

Again, the results furnish information only as to the constitution of the aggregate and not of the units of enumeration; for the unit is known and its limits prescribed before the enumeration. To fill the conditions of problems the things counted must correspond to this idea. They agree because they fall within the limits of this characteristic idea, but only so far as this is the case. As to other peculiarities their constitution may be very different. Everything counted as a unit must necessarily have other characteristics besides those in question. These other characteristics remain unknown quantities.

The addition made to our knowledge is simply that things which correspond to the given idea are found in a certain total, or possibly not at all in the aggregate. As to this aggregate, the field of the investigation, there is some new knowledge. But as to the things enumerated

there is a new insight, only in so far as they may be characterized by their presence in definite numerical relations in the aggregates investigated.

§ 72. ENUMERATION OF UNITS IN COMBINATION.

The problem of investigating the manifold characteristics of an aggregate from the number of fixed known and definite things which go to make it up, is rendered easier and more profitable the greater the number of things which may be counted and the more they may be brought into combination. This combination of the units of enumeration may be of several kinds.

The different units may be related simply in their juxtaposition. The enumeration finds the unit A (persons) x times, the unit B (houses) y times, and the unit C (horses) z times. The judgment is that x A, y B, and z C are in the aggregate.

The units can be chosen from their relations to certain qualifying ideas. The unit A can be counted as qualified by several characteristics; a (male) b (single). Then the result is that $y(A+a)$, $z(A+b)$ are found among x units of A. These qualifications may or may not exhaust the idea A. They may describe it from the different points of view: D (arable land) enumeration according to c (area), or d (value), hence $w(D+c)=v(D+d)$.

The choice of units may be made according to some supposed causal connection; *e.g.*, E (children), F (teachers). The result is, for u E there were t F. Whether cause and effect are here at work, and if so, how, is not determined.

The greater the number of possible relations among the units, as suggested by induction, and the more systematically the effort is made to bring out such relations, so much the more must the numerical data obtained add to our knowledge of the conditions existing in the aggregate.

§ 73. RESULTS OF THE ENUMERATION.

The results of the enumeration form, it is obvious, a description of the subject investigated. This description is more comprehensive when large and systematically connected groups of different units are counted. It is attained, however, when the enumeration is carried through with a single unit.

It must be recognized that such a description would be attained, either positively or negatively, if the observer, having before him a real field of view, and without any knowledge of it, should conceive a certain thing as a unit, count it without any definite purpose. This would be true whether he found it within the definite limits of the field or not. The fact that it is a conscious problem lends a meaning of the description which is the result of methodical enumeration. Essential premises are drawn from this conscious purpose, which cannot be separated from the process. This purpose follows necessarily from the fact that the methods of statistics are applied to fill out, if possible, some gap which inductive and deductive methods have consciously left open in the investigation of the multiplicity of actual existences. The aggregate investigated is known as a whole, as an individual or collective idea; and it is examined in its composition and internal connections, which are unknown.

This examination is not without plan. It is not directed toward any chance object, but the enumeration bears upon a unit chosen expressly with reference to its significance for our knowledge of the whole.

So simple and apparently superficial a process as the mere counting of certain things is only chosen because it is believed that some gain in knowledge will result from the determination of the number.

Statistics presupposes for its method a wealth of experiences and abstractions. The methodical description obtained by enumeration rests upon a basis of well-connected

ideas, which is to be further built up by the results attained, or to be better adapted by them for certain definite purposes.

Nevertheless, the peculiar barrenness and inflexibility of this summation is not to be mistaken. The question is not of this mere description, but of the real importance of the gain which these figures promise.

While the units observed correspond to real facts, these facts are determined beforehand by fixing certain well-defined characteristics, not by exhausting all. The only additional determination of the idea of the phenomenon which is obtained, is whether it is contained once, twice, more frequently, or not at all, in the aggregate. Here the size of this number is of decisive value.

Numbers and numerical successions have a certain gradation in themselves; *i.e.*, 1 is half as much as 2, and 40 ten times as much as 4, and so forth. But when in a certain aggregate A is found 10 times, B 40 times, and C 1000 times, or 20 a and 30 b among 100 c, these figures give obviously no measure of each other, nor in fact any comprehensible measure. Whether it is much or little is impossible to tell.

Empirically it may indeed appear otherwise, because in our general thought, and by recollection of numerous statistical enumerations, we have a number of hazy ideas as to the number of units (men, houses, goods) which were in a certain locality, and such ideas are used instinctively as measures, however deficient they may be.

The strict methodical process cannot rely upon such uncertain support. It must require that the true importance of figures be measured on an exact standard.

Such a standard might be obtained, one would think, by such an investigation of the aggregate as would show how many of the things counted it ought to contain, and must contain, conformably to its essential character, and in what degree it had the capacity or possibility of containing fewer or less. But this method is obviously that of the experi-

mental investigation of the aggregate with all its confused and changing complexities, with all the possible combinations of its elements, and their wealth of mutual inter-relations. Such an investigation may be said to be impracticable in principle. But if by any possibility it might appear actually possible, it could not be avoided, and in that case the statistical method would be rendered superfluous.

Under the conditions of statistical inquiry it is not possible to obtain a measure of the results of the enumeration of an aggregate, simply from the nature of the aggregate as apparent from the enumeration. With the demand for such a measurement the whole process reaches a point where it can no longer confine itself to the observation of the aggregate to be investigated, and where the total work of enumeration, however complicated, difficult, and extensive it may have been, is clearly recognized as only of a preparatory character.

B. PRINCIPLES OF STATISTICAL JUDGMENTS.

I. QUANTITATIVE RELATIONS.

§ 74. THE MEASURE OF QUANTITY FROM ANALOGOUS AGGREGATES.

The description of any investigated aggregate by a statement of totals obtained by counting its component parts, does not furnish any new information, unless we can form a judgment of the meaning of these sums. Whether a single unit or a number of units has been counted, is wholly immaterial.

A judgment on the number of observed phenomena must necessarily be one of proportion; that is, of a quantitative nature. Are the figures obtained to be considered large or small, and if so, in what degree? This is the

question which arises, and the necessity of some measure, some scale by which to judge the results, becomes obvious.

Statistics finds this measure in the comparison of the results obtained for similar units in analogous aggregates. These may be the fruit of previous or of simultaneous investigations.

The proportion which the different units of enumeration bear on the average to equal portions of the analogous aggregates is calculated, and the limits of the deviation from the average in both an ascending and a descending scale are observed. Thus, a measure is furnished with which to judge of the numerical relations of the units in any particular aggregate. What is meant by analogous aggregates scarcely needs explanation. It appears clearly that they are aggregates which considered as a whole come in the same category.

It is from this point of view possible to conceive of the number of analogous aggregates, changing, as they do, according to the problem in hand, as almost inexhaustible. The number may, however, be very small. In every case it is limited by the fact that we can actually use only such aggregates for purposes of comparison in which units similar to those of the aggregate investigated are or have been counted. Hence the measure of comparison must be drawn from a comparatively restricted group of analogous aggregates, and everyone using it must bear in mind upon what group of analogies the judgment is based.

The analogous aggregates are sufficiently known. They are not chosen from those manifold and confused characteristics, which are the subjects of statistical inquiry, but from those definite qualities which as a whole place them in some general class. Conceived as a whole, they must be judged by inductive methods, and it can be seen which have more importance than others for the comparison, and which, therefore, would constitute more valuable elements of the scale. This building up on information

gained by induction, is a very general and necessary element of statistics. The ideas of empirical thought are earlier than the questions of statistics. Inductive knowledge is only supplemented by statistics where gaps remain and further progress is barred.

The difficulty of the process does not lie in the often very limited sphere of analogies which can be used in forming the scale, but rather in the proper conception of the aggregates which in concrete cases may be looked upon as analogous.

§ 75. THE CHOICE OF ANALOGOUS AGGREGATES.

The choice of analogous aggregates necessarily depends upon their character as units. This depends in the first place on that particular conception of the aggregate as an entirety, required by the problem.

In this connection it is of course requisite that every field of statistical investigation may be generally conceived in its entirety as a single thing, whether as an individual or as a collective.

Evidently, statistical enumeration and the kind of description which follows from it are utterly incapable of characterizing the aggregate as individual or collective. All essential qualities of the individual, as well as of the collective, which are conceived as fixed, or changeable, relate to its organization as an entirety. But the statistical enumeration concerns only certain things among the various elements which compose this entirety and determine their number. If this number belongs to the notion of the entirety, then it is typical. But in that case it would be known already by means of general or experimental observations, and have found a place in inductive and deductive generalizations. There could no longer be any question of an element still to be found in an unexplored and changing complex.

Nevertheless, each of these complexes, with confused

and unknown elements, would have certain definite characteristics as an entirety. A State, a country, a harbor, a town, a section of the heavens, the monthly weather of a station, and so forth, have all general qualities that characterize them as entireties, so that other entireties falling in the same categories may be recognized. According to his notion a human being is known as an individual, but he is capable of conception as collective, at any rate as an aggregate. Numerous more or less changeable elements are to be observed upon him, as teeth, eye-brows, and freckles, which, for a medical or anthropological purpose, might be subjected to a statistical enumeration and comparison.

The idea, therefore, which is to be held of the investigated aggregate as an entirety determines the analogy with other aggregates. Particular attention must be given to the elasticity of the terms used, and the application of very different notions to one and the same actual entirety.

For each statistical problem the aggregate to be investigated can be conceived only in a single way. If that be changed the notion of the aggregate changes, and with it the problem. What are apparently two identical problems become two different ones. The reverse holds, and the change of the problem involves a change in the notion of the whole. If it be asked whether Prussia had a large growth of population during the last census period, the notion of the aggregate Prussia, and hence of all analogous aggregates, is very different, according as the question means, was the growth large as growth or large for Prussia. In the first place, Prussia is conceived as a civilized country, and the analogy attaches to that notion; but, in the second case, Prussia is conceived with all its peculiarities, and it has no analogy but itself. Here the task is simply to compare the aggregates of Prussia at different census periods among themselves.

Now, it is obvious that according to the nature of the

problem a limited group of aggregates, instead of many, may be sufficient to furnish a standard of comparison. It would certainly be sufficient in the first case to compare Prussia with the most important States with trustworthy statistics, and in the second case the Prussian figures for the last three or four decades.

The notion under which an aggregate to be investigated may be conceived for comparison may vary from a definite individual to a part of concrete existence, limited in time and space, but otherwise wholly unknown. Hence the choice of analogies requires acute distinctions, but is always possible. It is questionable how far comparison must be abandoned because an enumeration of like units in analogous aggregates has not taken place, or cannot be effected.

The problems of practical statistics are usually those for which provision for comparison can be readily made at the time of enumeration, or already exist in previous results. If these problems are to be solved within the limits of a single country, there are, as a rule, a great number of comparable aggregates for which exactly the same enumeration has been made. We have, on the one hand, the minor subdivisions of the country, as provinces, counties, even to townships, which are distinguished in the enumeration, so that each division is a separate aggregate, the sum of whose units may be compared with that of others. On the other hand, from year to year, or at definite periods, certain enumerations are made where like units are counted in the various subdivisions of the country, and thus a new series of analogous and comparable aggregates is obtained. For such questions which extend beyond the limits of one's own country, there are means of solution to be found. Most civilized States undertake, from similar necessities, similar statistical enumerations. But it cannot be denied that international statistics show some very bad gaps and other imperfections.

They are not caused by a lack of analogies, but by a

lack of sufficient agreement in units of enumeration. Like names for things counted may lead one to overlook that different States apply the same designation to units which are different for purposes of comparisons, because unlike in composition and origin. (O. Haussner, *Vergleichende Statistik von Europa*, 1864.)

Statistical problems are always incapable of solution when it is impossible to find a basis of comparison in analogous aggregates examined with reference to exactly the same units.

§ 76. REDUCTION TO COMMON TERMS.

A comparison of the analogous masses is not attainable directly. Some basis of comparison must be found by reducing the results to common terms. This reduction depends, like the choice of the units, upon the nature of the problem.

If the purpose of the problem is to determine the relative frequency of the occurrence of the phenomena, the common terms must be in measures of time and space for the various analogous aggregates. Hence there must always be an equal measure of time and space—thus, for 1 square mile in 1 year x wheat. No limitation of space is necessary when the aggregate has changed in time only and not in space (Province A, 1 year y taxes), or, when in an observation of movement, the place is a point from which the number is counted (Canal B, 1 week z vessels). In like manner the time needs no limitation, when the condition in all parts is recorded at one moment (State C, 1 sq. m., x inhabitants, y houses, z arable land). In all these cases time and space are bases of the comparison.

The measurement of time has naturally few difficulties. On the contrary, the measurement of space requires often very extensive operations of survey and triangulation, which statistics are not in a position to carry out. It has to presuppose the existence of this, and expects to find it completed, just as much as the measuring of the qualities

of the units. The survey and triangulation of territories, or calculations which are not always exact from the geographical boundaries, furnish the necessary data in this matter.

The number of things enumerated for each common term is found by calculating what part of the entire aggregate it forms. The same proportion of the total number of units must fall upon this term. Therefore, for these common terms the most various units can be shown side by side with all their distinctive differences. This division of the total is simply one of calculating proportions, and would never justify the conclusion that a similar distribution in the density of units occurs in the real parts of the field.

As the aggregate is to be measured by mutual relations of the units, the reduction to common terms takes the form of the reduction of various units to an equal quantity of one unit, and, as a rule, percentually (for 100 units A there are x units B). Any unit can be used for this purpose. Specifications of the main unit are hereby rendered much clearer in their relations to the central idea. Sometimes for such combinations an equal period of time must also be drawn into the problem (as for 100 inhabitants y births in 1 year). In like manner a restriction of place may be necessary. But here the percentual number expresses simply an average, not at all that in the aggregate or its parts, or anywhere in particular, for 100 units A there are x units B in nearer or more definite relations. The reduction to common terms which always has the same purpose, becomes a very extended calculation when it deals with many distinctions and differently graded qualities. (Ernstatistik, Bd. I. d. Stat. d. Deutsch. Reiches, p. 110.)

§ 77. SERIES, MAXIMA, MINIMA, AND AVERAGES.

The results obtained by reducing the results of enumeration to common terms always furnishes a series admitting

of comparison. The figures for the same unit show in which aggregate relatively the largest and in which the smallest number of units is to be found. Arranged from minimum to maximum, they show the order in which the degrees of intensity progress. Arranged according to the geographical position of the masses, they show the intensity in its distribution, and arranged by their succession in time, the fluctuation of their historical development.

The fluctuation appears either as regular growth or decrease, or as an oscillation which, though changing, returns to similar maxima and minima.

Two or more such series placed beside each other, and thus showing a comparison of different units, may be examined as to the similarity or dissimilarity of their courses, whether they oscillate in the same or opposite directions, and whether the maxima occur positively or negatively at the same point of the series or not. (Compare Appendix III.)

For the fluctuations of a single series we find the measure in the average. The average expresses the mean figure, the mean importance of all the fluctuations of the series, so that the individual figures extend beyond it, some negatively, some positively, in equal proportions. The average cannot be gained immediately from the series itself. It must give the mean value of the totals of all the aggregates, and cannot, therefore, be calculated from the ratios, but only from the absolute figures, the sum of all the units of enumeration of all the aggregates compared, divided by the sum of all the quantities used for the reduction to common terms. For the comparison of several series, there must be an average for each to serve as a basis of comparison. (Lexis, quoted § 16, and Marey, quoted § 57.)

§ 78. RESULTS FROM QUANTITATIVE JUDGMENTS.

From such series of relative numbers we can form a quantitative judgment, that is to say, a conclusion as to

the greater or lesser measure in which the things determined upon as units of enumeration are present in the aggregate investigated.

The more numerous the series the clearer is our insight into the subject, since we know the relations of a larger number of component things. And the more varied the relations among the things are, their grouping as genera and species, as well as other relations, the greater is the definiteness which the aggregate assumes. We also know in what relations these things occur in other aggregates, which, however variable, may be regarded when considered as units as belonging to the same class as the aggregate which has been investigated.

The things themselves are not examined. We find simply the frequency of their occurrence, their quantitative distribution in certain limits of time and space, among other things which remain unknown, but which fill out the space. Thus a certain general measure of their mutual relations is determined, in a somewhat summary manner, it is true, but in one not to be reached by other methods.

Thus the aggregates themselves are better known and are more capable of comparison. As they agree in those essential items which characterize them as individuals or as collectives, it is very important to find that other elements, which appear fluctuating and unstable, can be measured, at least as to their intensity and probable mutual relations, by definite measures, and in relation to similar aggregates.

Thus the first step of statistical judgment, *i. e.*, the judgment of quantitative relations, is complete. It is the indispensable basis of any further progress. By far the greater part of the statistical activity whose results reach the public and become common property, does not go further than this first step in the knowledge to be gained from statistical enumerations. Ponderous statistical publications of official bureaus deal in the main with this class of description. It

is rarely that an investigation steps beyond the limits of this quantitative judgment. This is all the more explainable, as this is the field in which, as much as possible for empirical knowledge, exact results and conclusions may be reached.

All further conclusions based upon these results, assume unavoidably a more or less hypothetical character. Nevertheless, it would be quite impossible to confine statistical investigations to quantitative judgments, even if we should follow the most restricted conception of the limits of statistical science. The final and conclusive utility of this primary requisite of methodical statistical investigation is certainly not to be found in the rich materials that it presents, but in the conclusions to be drawn from them.

II. CAUSAL CONNECTIONS.

§ 79. REQUIREMENTS AND LIMITATIONS OF JUDGMENTS OF CAUSALITY.

The next step in statistical investigation after the details of the quantitative relations must be an effort to explain the reasons for the numerical relations which have been discovered; in short, their relations of cause and effect. It is in the nature of our reasoning to suppose a cause for every effect, and to seek to find it. All phenomena are subject to the rule. For every phenomenon we think at once of a cause and try to find it; or, in other words, we seek a satisfactory explanation of the phenomenon. While our perceptions have the form of descriptions, the interest we take in them is the effort to determine causality.

To demonstrate the causes empirically is very difficult. We are conscious of causes only in our own actions. In the external world they appear only as succession in time, and whether A is the cause of B, which follows it in point of time, cannot be maintained without danger of error.

For individual cases, therefore, the proof is carried out by experiment. The object is placed successively under one influence to the exclusion of all others, and the consequences observed, until it appears which influence brings about the effect.

Statistics can make no such experiment with the phenomena of aggregates with which it deals. In the complex aggregate the most varied and opposite causes must always be at work. Statistical methods cannot expect a knowledge of all the things in the aggregate, and cannot, therefore, attempt an investigation of all the causes in operation. In the judgment of causal connection, it is limited to certain phenomena, the frequency of whose occurrence has been numerically measured. It can only ask what causes have produced these relations, or what effect may be traced to them.

The process by which the answer is made resembles somewhat that of experiment. We take up in succession the possible causes. We may suppose, at first, the most probable to be actually operative, and look for indications whether or not this is the case. If not, each probable cause must be examined successively, until one appears to be sufficiently well founded. If the supposed causes are exhausted without success, then there remains simply the same result as in a fruitless experiment, that, of all conceivable causes, none seem to be at work.

§ 80. DISCOVERY OF CAUSALITY THROUGH FUNCTIONAL SERIES.

Statistical methods can only look for indications of the causal connections of two phenomena, in the mutual relations of the quantities in which they are found. Their frequency in the aggregate investigated is not alone sufficient for a demonstration.

This is expressed by two figures which are otherwise unrelated. For any judgment as to the meaning of these

figures it is indispensable to compare the figures which have been observed within a group of analogous aggregates for the same units.

Since the cause produces the effect, the cause and effect must be weaker or stronger together. Whether an increase in the intensity of the cause shows itself positively or negatively is indifferent. Our idea of a sufficient cause is based upon its functional relation to the effect.

The proof of causality can only be given in the course of two series of relative numbers: one which shows the measure of the cause, the other the measure of the effect, in the aggregates compared. If the effect A is really the result of the cause B, the relative number of the units A must in all these aggregates vary in the same or opposite ratio as that of the units B.

But if such a relation, with the exception of such deviations as may be explained, does not appear, the supposition of causality is to be rejected. On the other hand, the agreement of two series is not to be considered an absolute demonstration of the causal connection of the phenomena. It is only a presumption of such a connection. Both may be the effects of the same cause, or, indeed, numerous combinations of the most divergent causes might lead to a similar final resultant in each series.

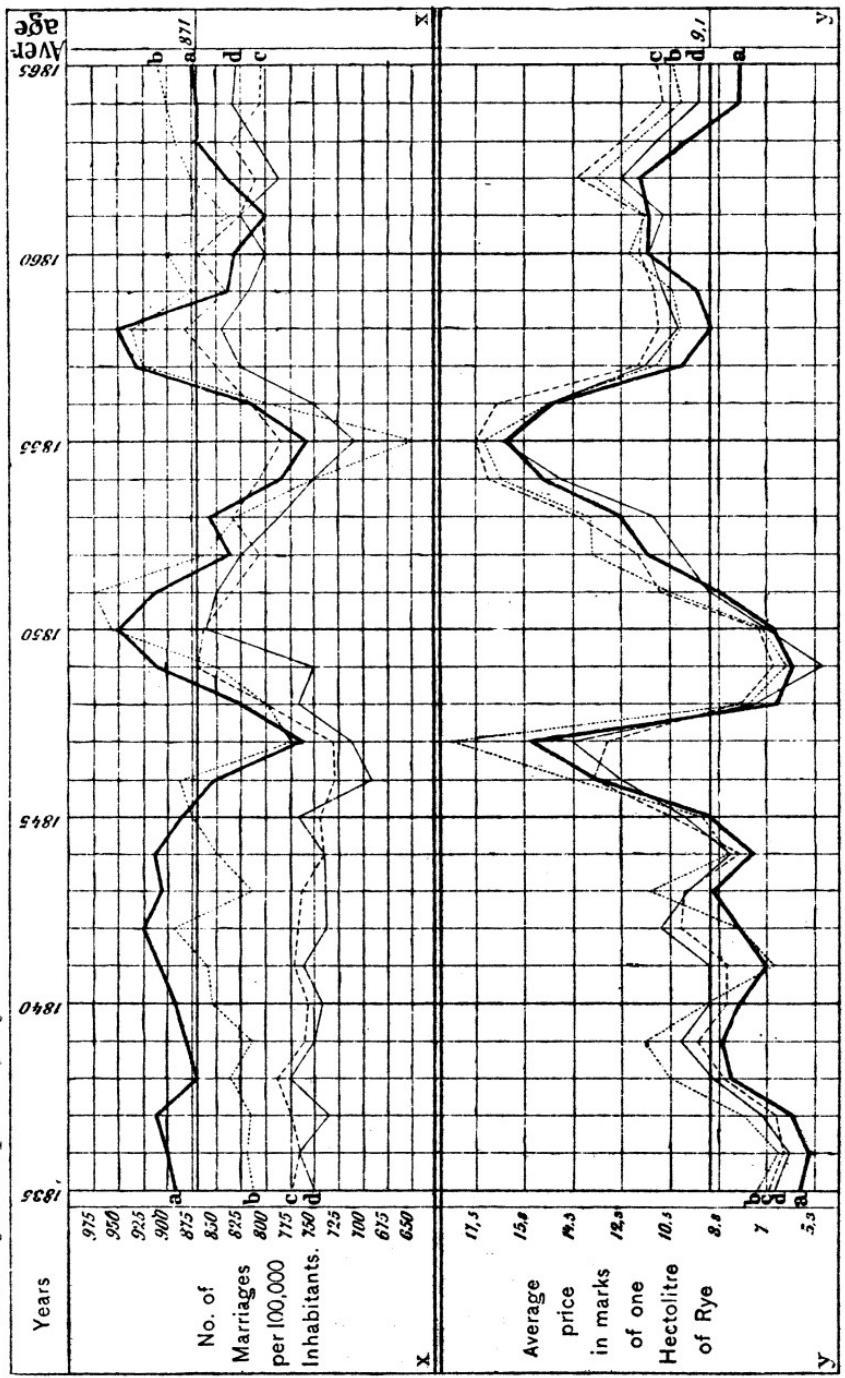
Hence, to decide as to the nature of connection between agreeing series we require still further evidence.

Such evidence can be found, as in the choice of analogous masses, only by the use of knowledge which has been gained by the inductive processes of experiment and observation. Numerous causal connections between single things, individual as well as collective, are sufficiently well grounded to be generally known. From such knowledge one may by induction or deduction presume the effect of a given cause which is the most immediate and probable.

In order to know the cause whose effect is expressed in the series A we must fix our attention upon the cause Z,

Marriages and prices of Rye.

Appendix III. (to § 80).



a, Prussia; b, Saxony; - - - c, Hanover; —— d, Oldenburg.

Based on Statistics of the Grand Duchy of Oldenburg, Part IX, 1870, Appendix I.

which inductive knowledge tells us is the most immediate cause of which A is the effect. We then note the series of the phenomenon Z in all the analogous aggregates to be compared. If the series Z is present among the enumerated units, it is easy to decide whether the functional relation of the two series permits us to suppose a causal connection in the case. If the series Z has not been counted, this must take place, or a sufficient substitute, which can take the place of the series Z, be gathered from the enumeration.

If it appears that no functional relation exists between the two series we cannot assume this cause to be of influence, even though under ordinary circumstances it may be the most probable. We must rather look for the cause Y, next in order of probability, and apply to it the same process, and so the successive probable causes, X, W, V, T, etc. Thus we continue until we find a correspondence between the series with one of the causes, or else we must abandon the hope of establishing the real cause from among the probable ones which we know. In this case the negative result is undoubtedly of great value. All presumptions are cut off which seemed most reasonable, and this negation may serve for the solution of exceedingly important problems.

The result may be that another series, B, which no one supposed to be connected with the causes, is now found to be important through excluding the seemingly more immediate causes. It may be apparently improbable and difficult of comprehension, but must, when all other connections fail, be recognized as the fact in which the forces at work find a clear expression. An example is given in Appendix III.: marriages and the prices of rye.

§ 81. SYMPTOMS AND THEIR APPLICATION IN DETERMINING CAUSALITY.

In finding an explanation of cause and effect we do not have all the elements of the solution in the problem itself, as in the case of the quantitative relations of the phenomena.

The necessity of introducing certain phenomena for comparison appears in the course of the proof. The investigation of causality does not look for the scattered and characteristic features of a picture, but the confirmation of a chain of more or less obvious or obscure conjectures, none of which can be disregarded without fear of error. It is purely a happy accident if all the figures necessary for the comparison are in existence or can be obtained. In case no enumerations have been made for analogous aggregates of units essential to the proof, or are wholly impracticable, something else must, as pointed out in § 62, take their place. We thus arrive at the question of so-called symptoms.

These symptoms are indications, but they cannot, as demonstrated by § 65, be found in qualities or abstractions from conditions. Statistical methods cannot say that things (money, capital) are symptoms of qualities (rich), or of abstractions from this quality (riches). Only persons can be rich; whether they are rich and in what degree can only be determined in considering persons, and to say what capital constitutes riches, makes sense only in connection with persons. Capital expressed in money can be counted, but it shows riches only in the relative possessions of persons.

A symptom means in statistics something capable of enumeration, which shows by its number that something may be inferred in a corresponding number, or one to be estimated or calculated. For different degrees of rich persons, whom it would be very difficult to count, we find a symptom in the returns of persons liable for income or

property taxes. In the statistical inquiry as to causality one of the most important problems is the choice of these symptoms. Gioja (see § 46) called attention to this in an instructive way.

On the other hand, there are no general rules for the proper choice and use of symptoms. One thing can never be replaced exactly by another. The acuteness of the judgment must suffer in some degree. At the same time the application of the theoretically best symptom is oftentimes quite as impossible as the use of direct results of enumeration. A suitable choice must remain a matter of a clear and keen conception of the special problem, and of all the resources which may be drawn upon for its solution.

Some misgiving may arise from the need of symptoms, but it cannot be overlooked that they bring an important element of flexibility and variety into the methods of statistics. By means of one unit replacing another the results of enumeration may be utilized for conclusions, which would have required in the original problem or in others an enumeration under totally different circumstances.

III. PROBABILITY AND REGULARITY.

§ 82. THE CONCEPTION OF PROBABILITY.

The hypothetical demonstration of causal connections in statistical aggregates furnishes the basis for the solution of a vast number of practical and scientific problems. It is, moreover, indirectly of peculiar significance for statistical methods. The insight into causal connections renders possible a judgment of probabilities and the expectation of regularities.

This further step in statistical knowledge opens up a wide possibility of solving many problems, even quite complicated ones. By its means conclusions may be formed as to the quantities in which qualities, which have not been enumerated, are to be found in the aggregate to

be investigated, and which may, therefore, be expected with adequate regularity in similar aggregates. What we know as probability is based upon the idea that what occurs regularly in our experience under certain circumstances is very likely to occur again in the same way if the circumstances do not change. Phenomena which have been demonstrated to be the result of a certain combination of circumstances, even though some are not wholly known, may be expected with greater probability than any other phenomena from the same combination. This line of thought is distinctively causal. The circumstances are the causes, which being uniform must produce uniform effects. If the circumstances cannot by any possibility change, and permit only a limited series of possible results, a calculation may be made of the degree of probability of each of these results. This is the idea of the calculation of games of chance, and of Bernouilli's principle of Probability (§ 15).

This principle is expressed as follows: "If a large number of cases have been observed of several events which have a certain probability, the number of cases in which each event has occurred will approach very closely to its probability, and this more and more precisely the more extended the number of observations." Bernouilli, however, limits this rule, inasmuch as when the number of possible events is infinitely great, that is, when the number of events is not limited, but may expand indefinitely, the principle does not hold. In many statistical problems, as in the calculation of death tables, the series of possible cases may be regarded as complete, on account of the infinitesimal number of those dying at very advanced ages, and Bernouilli's principle may be considered applicable.

As a rule, however, the form of the calculation of probabilities, as applied by statistical methods, undergoes a two-fold modification. On the one hand, it results from the nature of the aggregates to be investigated, that they include an infinitely great number of different possible

phenomena. On the other hand, it is only exceptionally that statistical problems require the degree of probability of a certain phenomena, but rather in what quantity or what relation a certain phenomena may be expected. This is a reversal of the problem which simplifies it, and renders less important the lack of a definite limitation of the phenomena.

In all cases it is clear that a large series of aggregates is much more likely to comprehend all the possibilities of the occurrence of a phenomena than a small series. It is, nevertheless, just as little to be doubted that without limitation even the largest series does not give any absolute security that all possibilities are actually contained in it.

In order, therefore, to obtain a definite numerical expression by a calculation of probability instead of by enumeration, statistics must make use of a fiction. This fiction expressed in logical terms is that in the series of enumerated aggregates which are compared, the possibilities in question are to be regarded as exhausted. Actually this supposition has a concrete justification in the fact shown in § 75, that the analogous aggregates are chosen with knowledge of and reference to their peculiarities. Statistical problems employ this fiction so extensively that the sphere of comparison becomes frequently very narrow. In case of necessity the probability is often judged from the results of a single enumeration; *i. e.*, the possible variation is estimated.

But, as a matter of fact, the difficulties and the interest of these problems do not hinge so much on the questions, which cases are possible, and hence may interfere with the action supposed, but they rest on the doubt whether known possibilities have been correctly judged in their causal connections.

§ 83. SUPPOSITIONS BASED ON SIMILAR CAUSALITY.

The process of assuming by probabilities the number of things which have not been counted is based wholly upon the idea explained in the foregoing section, that in the aggregate to be investigated the same causes operate as in the analogous aggregates. Like numerical relations under dissimilar conditions would be highly improbable. These conditions of the conclusion as to probability cannot but exert an influence upon the choice of analogous aggregates. The analogy of the aggregates is determined by the same conception, be it broad or narrow, of them as entireties, either individual or collective, presupposing always that the distinctions have been drawn with care and attention. Whether or not the analogous aggregates have actually been subject to such similar circumstances, that the causes at work from which the probable figure is deduced have been actually identical, is one of those questions for whose answer statistics must draw upon general empirical knowledge. Here it is necessary to consider, according to the nature of the problem, the different influences which may be at work in contemporary history, in political or economic life, and in the processes of nature.

Such influences may often be determined statistically, and be observed in the search for cause and effect. Whether this takes place depends largely on the time and cost involved, and on the importance attached to the question. Influences which are generally operative are to be regarded as causes, since they affect the relations of the parts to the whole in the same way in all cases. When it is known that influences are at work whose effects are partial only, the aggregates in which they occur must be withdrawn from the examination. For it is preferable to know positively that like conditions exist among the aggregates compared than to extend the comparison to a great number of them.

In all events no effort should be spared to ascertain whether special influences have been at work in some of the aggregates considered as analogous, or indeed whether the aggregate to be investigated has not been subjected to some unknown influence, which would render unreliable any conclusion upon the basis of probabilities.

Nevertheless it would not answer to indicate these possibilities of error by expressing the probability in hypothetical or indefinite numbers. The probability would be useless for any scientific purpose by such uncertainty. On the contrary, any doubts as to the certainty of the result must find expression with sufficient explanation in a critical examination of the margin of error.

Employing the fiction already named, that the aggregates compared exhaust all the possibilities to be considered, it is an easy matter to obtain the most probable number. In one case the desired phenomena has not been counted in the aggregate to be investigated, but in the analogous aggregates, whereas the series of causes or effects corresponding to this phenomena have been enumerated in all. Here the phenomena in question will bear the same relation in this series in the aggregates to be investigated as it does on an average in the analogous ones. In another case the series of causes and effects have not been determined for the aggregate under investigation, but it is supposed that like circumstances affect this and the analogous aggregates. Here the desired result would be the average of the analogous aggregates reduced to an equal size with that investigated.

When a series of enumerations for analogous aggregates is at our disposal, and the probability may be calculated from their average, it seems advisable to note the possible deviation of the actual number from this average. In the most extreme case the deviation can be supposed to equal the actual extremes in the series. When, however, we desire to find the probable deviation on one side or the

other, we can calculate the mean of the deviations from the average on each side.

The extensive application of probabilities has led to certain points of view which require more precise explanation, and a correct estimate of their relations to the whole field of probability. They are the so-called law of large numbers and the so-called regularity of seemingly voluntary actions.

§ 84. THE SO-CALLED LAW OF LARGE NUMBERS.

The so-called law of large numbers has often been treated as some mysterious force peculiar to large aggregates, through which large numbers establish regularity. The conception is more correct, that in a large number the actual relations are more accurately expressed than in a small number, and hence probability may be concluded much more safely from a large number.

If there were any necessity for the phenomena found in large numbers, it would be equally well expressed in the small ones. And if this necessity were recognized, it might indeed be developed analytically and its results calculated. But this is not the case. The reason why the character of an aggregate and the circumstances at work in it are determined more readily from a large number of phenomena than from few observations, is simple. Every aggregate is composed of various things, and with general uniformity there may still be some exceptional irregularities. If a small number of phenomena, that is, a portion of the aggregate only, is considered, it cannot be estimated whether accidental exceptions, causes of error, and unusual influences may not be entirely lacking or especially concentrated in this portion, and hence any conclusions from it as to the erroneous nature of the whole. In a large number such exceptions must appear in nearly the same relations as in the entire aggregates. A single case can only be either regular or

exceptional, while in the whole regularities and exceptions assume their proper proportions. A large part approaches the whole much closer than a small one.

In a derivative sense this might be called a law, because it is a truth uniform under all conditions, and a rule for the judgment of probability. But the rule has obviously no connection with the relations of the facts themselves, but only with the formal process of determining them. The phenomena and their effects occur in the same way whether few or many are observed. The law of large numbers has no more influence upon the facts than enumeration itself. It is not a rule of causality, but simply a rule of perception.

§ 85. THE REGULARITY OF SEEMINGLY VOLUNTARY ACTIONS.

The doctrine of the regularity of the seemingly voluntary actions was formulated by Condorcet, and has exercised an important influence upon the theory of statistics since the time of Quetelet (§ 51). It is based upon the fact that certain groups of human actions dependent upon the will show approximately the same numerical relations in their distribution in time and space. On this basis there has been postulated more or less emphatically a law governing with constraining force the will of man. In its elaboration this has led to the idea of laws ruling the entire historical development of the human race, capable of the same calculation as those governing inanimate matter. With this in view, some have broadly denied human liberty and moral responsibility, while others have admitted the freedom of the will for individuals, but denied it for large groups of human beings.

A law governing the will is in fact incompatible with responsibility to the individual conscience, or toward one's fellow-creatures or the State. Such a law would dissolve all religious convictions, the State and civil law. It must

either conform to the laws of the State, in which case the laws would be obeyed without being upon the statute books; or it must be contrary to those laws, and in this case they would be brushed aside, but without criminality or redress.

Whether or not such a position is admissible from an ethical, psychological, or anthropological point of view, cannot be decisive for statistical theory. If it were provable by statistical means, the theory of statistics would have to be shaped in accordance with so weighty a fact. No end could be more worthy of statistics than to prove by its critical methods a definite cosmological system.

An investigation of the facts shows, however, in the first instance that such uniformity of the numerical relations as would justify the conclusion of a constraining law cannot be conceded. Suicides, which are looked upon as the most decisive proof of the law, have, between 1836-1875, in quinquennial periods, varied per 100,000 inhabitants as follows: Belgium, 3.65-7.01; Austria, 4.60-11.93; Germany, 4.83-9.16; England, 5.57-6.69; Sweden, 5.66-8.49; Norway, 7.33-10.90; France, 7.62-14.55; Prussia, 10.28-13.28; Saxony, 16.00-28.10; Denmark, 21.40-28.60. If from these figures it is deduced that among 100,000 inhabitants 100,000 suicides would have been possible, and only 3.65-28.60 have occurred, this would be a reasonable approximation. But if it is to be formulated as a law that among 100,000 inhabitants 3.65-28.60 must commit suicide, we cannot accord our assent. Even if in each State only the given figures were used, and the minima and maxima represented always a steady progression, and not ups and downs, there could be no conclusion of constraining force. This becomes more and more evident in comparing lesser areas and periods.

Similar results are to be obtained from a detailed examination of the other means of proof, which relate chiefly to the illegitimate births and various kinds of criminals.

The motives of the actions whose voluntary character

has been questioned are in most cases known or capable of determination. They are fully conscious, and often contradictory, according to changing circumstances and situations of the persons. Certain motives of suicide and crimes, and also certain modes of carrying them into execution, increase with good or ill fortune, necessity or plenty, the quiet or disturbance of contemporary events.

A law with different causes cannot be a natural law. The force of a natural law is permanently and uniformly inherent in all natural bodies. Its effects may be rendered difficult of perception by the action of opposing forces, but they cannot be changed. A law of nature cannot affect the aggregates and not the individual; it works unconditionally in each individual and in the aggregate only by virtue of this fact. Death is based upon a law of nature, and possibly also the fact that twenty-one boys are born for every twenty girls. But the natural law must be operative in every death and every birth, and not simply in a certain number of cases.

Not a law of constraining force, but simply a similar combination of many and varying causes, is the reason for the approximately similar phenomena. But this basis is insufficient to support a doubt as to the existence of the freedom of the will. The expression is unhappily chosen, for the act of willing is always conceived as free. If external law controlled without opposition the will of man, his existence as a rational being must cease. Numbers do not control his will but only his action. No one has claimed for man entire freedom of action. The sphere of free action is always very narrow. It is limited by multi-fold external circumstances, by the power of environment, by individuals, by the State, by nature, as well as by the powers and means of the individual himself. But for free and responsible choice a broad field in the wide possibility of decision remains open. The scope of the latter is shown not only by the difference between submitting to another's will and preferring to destroy one's own existence, but

also in the inexhaustible modifications in the mode of execution and the ethical value of actions which differ but slightly.

Finally, it cannot be overlooked that all the figures relied upon to overthrow the freedom of the will are drawn from civilized peoples. These live under like conditions, and it is not remarkable that there should here appear a similarity of the spontaneous actions of individuals.

In itself human nature includes the uniformity of body and mind of the species; in civilized States there are added similar classes of population, similar distribution of occupations, of education, and of income, similar principles of law and administration, similar protection, and similar perils; and within the like classes, like views of life, like needs, like pleasures, and like habits.

It cannot be disputed that under all these similar conditions like phenomena are more probable than unlike, and hence great disparity would appear according to all presumptions much more inexplicable than the existing uniformity with its slight variations. These approximating results, which vary slightly according to the circumstances, appear much more natural as the result of the liberty of the will than as a consequence of constraining law.

Statistics finds within its own limits a satisfactory explanation of existing regularities, and does not need to borrow metaphysical or materialistic premises. On the contrary, it appears that the reversal of the law of probabilities leads conclusively to the same solution of this question.

If the probability of similar averages is based upon the causality of like circumstances and relations, the conclusion is unavoidable that the uniform or uniformly changing figures of moral statistics must necessarily be founded upon a basis of uniform conditions and developments. One can say that the regularity of voluntary actions is

explained most simply by concluding, from the fact, the same factors from whose existence the theory of probability would assume the fact.

§ 86. STATISTICAL REGULARITY.

Discarding all fantastical ideas of a law of large numbers or of unfreedom of the will, the regularity of phenomena is none the less a most important subject of statistical theory. "Rule" is our expression for the uniformity and arrangement of a number of analogous phenomena. Conformity to the rule is regularity. Hence we may properly call regularity things based upon analytical rules calculated mathematically, or uniformities following with necessity from natural typical premises (*e. g.*, that there are twice as many hands as men, or married persons as marriages). In the same way the necessary effect of a certain cause, or the same measure of effect with the same measure of cause; also the example of Sigwart (§ 58), when an object is found in relatively equal portions in the smaller sections of a State as in the whole, constitute regularity.

These are not statistical regularities. Statistics does not investigate uniformities that are calculable because necessary, nor such as are typical or accidental. These are either known or else occur so that no more interest attaches to their uniformity than to their difference, as long as no conclusion is permitted that a similar number will be found in the next aggregate not yet observed.

Statistics seeks regularities with the same end in view as in enumeration, *i. e.*, of gaining knowledge of the confused and unclassified aggregates of concrete existence. Regularity has value for statistical methods only as expectation. Expectation depends, like all probability, upon the knowledge of causal connections. This knowledge may be based upon proof or it may be assumed, possibly erroneously, but must always be conscious. Sigwart's example (§ 58)

is applicable statistically only where there is an object in view.

Conclusions as to regularities in statistics are conclusions as to probabilities, and are in a way an extension of the principle. Probabilities generally attempt to discover from the results of enumeration of analogous aggregates a number which in a special case has not been found by enumeration. Statistical rules, on the contrary, look for the numerical ratios for certain phenomena which may be expected for all aggregates, including those analogies which have not been observed. This inquiry cannot be conducted in any other way than by determining through enumeration the ratios of numerous analogous aggregates, and taking an average which may serve as a rule.

The application in a single case is now much simpler. If the rule is once ascertained, it is no longer necessary to compare the figures for all the analogous aggregates, but simply to examine whether the aggregate is actually analogous to those for which the rule is already ascertained.

This reversal of the procedure facilitates so much all statistical efforts, that it is one of the chief aims to find such rules. If rules are established for special causes and special peculiarities of the aggregate, the scope of the rule is thereby limited. In applying it one must carefully inquire whether all these peculiarities and special causes are present in the aggregate. Such special regularities may be most frequently found in dealing with the same aggregate, but at different periods. In such a case the non-applicability of the rule would not be without value, as it would point to variations of certain causes. But the more special the causes upon which the rule is based, the more they are apt to be variable and the more limited is the sphere of analogies. Hence the more difficult it becomes to find a sufficient number of analogies from which to gain the rule itself.

The rules are taken, therefore, largely from such relations as appear to be the result of the combined influence

of manifold causes known only in a general way, such as the influence of human society generally or certain conditions of existence or of occupation, of rural and city life, etc. It is chiefly in the field of the statistics of population that such rules have been gained. In particular might be mentioned the group of voluntary actions and moral statistics, and also births, still-births, plural births, marriages, deaths, age classes, expectation of life, sickness, physical strength, causes of death, civil condition, occupations, and other conditions of human life in which similarity of the phenomena are observed, as in the voluntary actions, and which furnish proofs of the influence of common circumstances leading to like results.

When the rules are of such a general character it is much easier to apply them to the analogies. They have a double use. In each new problem they seem a probable solution, and when, on the other hand, these conjectures can be compared with the results of actual enumeration in a given case, they lead one to inquire the special causes of the deviations.

C. THE FORM OF THE PROCESS.

§ 87. THE STEPS IN THE STATISTICAL INVESTIGATION.

The minute consideration of the steps in the methodical statistical process has shown its strictly logical character, and the symmetry of its principles.

The leading thoughts expressed in § 62 are the basis of the special phases of the mode of investigation seen in §§ 63–86.

These rules are developed from a critical examination of the value of our experience. We may endeavor to form a conception of a concrete aggregate; we may attempt to compare it with what we have observed, or with the more or less vivid descriptions of others, but in

the final result we have only a conception of it as a collective idea. In it float various elements that we cannot perceive. We may notice details; yet, however important they may appear to us, according to the subjective bent of our minds, they remain unrelated to the aggregate and purely accidental.

This defect of our faculty of observation can only be overcome by organized effort. This is to be found in enumeration, essentially the most general form of measurement. With a definite unit of enumeration the entire field of possibility is measured and the actual phenomena ascertained. The idea of gaining by systematic enumeration some knowledge of the confused and changing elements of an aggregate gives a fixed and definite course to the entire process. A deviation from the course involves an error.

The enumeration has for its objects certain concrete things already defined. One or more units may be counted, and, indeed, distinct measures for the various units may be adopted. This will furnish a somewhat richer and more detailed result, but does not manifestly affect the mode of investigation. The enumeration must not take place within ill-defined limits of time and space. The aggregate examined must have the appearance of a compact definite concrete object. The results of the enumeration are simply the totals of the things counted. These give some description, yet not such a one as would enable us to say that the totals were large or small. Such a judgment is the important thing attaching to the number gained. Other judgments could only be analytical ones derived from the ideas underlying the problem, and could furnish no contribution to our knowledge.

To reach a judgment we must compare number with number, totals of like things found in analogous aggregates; *i. e.*, such as have the same character for the purpose of the inquiry. It is not contended that this similarity exists concretely either for the units or aggregates.

Similarity exists only in the conception, in the characteristics necessary to it. All other characteristics of aggregate and unit can and must be very different. The comparison teaches how many things of the same kind are to be found in a series of aggregates of the same kind.

The next result is the idea of variability, or the limits and relations of the fluctuations. Theoretically it would be erroneous to attach the variability to one aggregate. One is easily induced to this by the impression that an aggregate observed at different periods is really the same with slight changes of certain qualities. According to § 65 a thing to be counted cannot be the same when an essential character changes. And so an aggregate is not the same in which the number of units to be counted changes or may have changed. It requires two enumerations to find out a change, and for each enumeration the aggregate is different. The analogy may be closer when it relates to aggregates of different periods of time than to those locally distinct, yet theoretically they are as different in the one case as in the other. The idea of the aggregate itself, a confused and changeable complex, inaccessible in its inner details to inductive and deductive processes, is contradictory to the notion that it might be considered as unchanged in the period between two enumerations. All variability which is observed statistically relates not to one and the same aggregate, but to analogous aggregates, those of the same kind. It is simply the different frequency of the things whose proportions describe the aggregate more explicitly.

Tracing further the statistical process, we find for all aggregates similar phases in forming judgments. The sums, when reduced to common terms, *i.e.*, the possible differences in the occurrences of the phenomena, furnish a measure for the quantitative judgment of the results. This judgment is limited to the aggregates compared. They cannot furnish a general standard. It is a standard, how-

ever, suitable for the purposes of the problems, inasmuch as the aggregates compared are sufficiently well known indirectly in their general character.

On this quantitative judgment we can solve the problem of causal connections upon the principle that the greater the quantity of things acting as causes the greater positively or negatively the number of things to be regarded as effects.

Like effects follow from like causes, and from the knowledge of causal connections it is but a step to the conclusion that the average relation between the quantities of cause and effect in analogous aggregates is to be looked upon as most probable in the aggregate under investigation, and further that from the quantity of cause one may calculate the quantity of effect and *vice versa*. This conclusion is so strengthened by numerous instances which confirm it, that regularity becomes one of the premises of statistical argument.

§ 88. PREDOMINANCE OF STATISTICAL MATERIAL.

The sketch of statistical processes given in the foregoing section seems to have but little connection with ordinary statistical work. Most statistical investigations show only portions of it.

This is accounted for in the purpose of the most common phase of statistical work. Nearly all statistical bureaus are engaged in preparing matter for work on problems of the future. This is true of official bureaus, of corporations, and scientific institutions, perhaps in a still higher degree of private undertakings.

It is in the nature of statistics that for special problems and sharp distinctions the analogy between the aggregates must be drawn very close. Hence for detailed investigations aggregates differing only in time are preferable to those differing in space. It is manifest that an aggregate remains essentially the same at different periods, however

it may vary in its parts. The fixed elements remain the same, and hence it is peculiarly adapted for tracing causal connections by statistical methods.

Questions which arise from the practical necessities of administration are apt to be such as cannot be solved at once by observations and enumerations. They often deal with the changes which have taken place in the field of the administration, and can only be answered by comparing past and present conditions. The investigation requires constant or periodical observations in order to determine numerically the changes which have occurred. In all statistical bureaus a system of providing for future uses has been developed, which far exceeds in extent actual solutions, and forms the main contents of the publications described in §§ 50 and 75. At each point there are certain data which it is considered necessary to collect continuously or periodically. In no case do they approach in comprehensiveness the field which the Statistical Congress regarded proper for such statistics. (Appendix II.)

The necessary restrictions of the field have already been noticed (§ 50). The great bulk of statistical publications belongs to the class of preparatory statistics.

It is profitable to establish rules as to what material is the most desirable under given conditions. Such rules might be called rational statistics. But statistical practice, with its changing needs, is rarely dependent to any great degree upon such a theory. Not only in different countries, but also in the same State, objects of enumeration and the treatment of them are only too often changed according to circumstances.

But whatever view may be held in the matter, it cannot be doubted that this phase of statistical activity brings about a peculiar conception and treatment of the material. This must be examined, and its relation to the method of solving statistical problems explained.

§ 89. THE ELABORATION OF SYSTEMATIC STATISTICS.

The comprehensive labors of statistical bureaus which collect data in all fields of statistics for future use may well be termed systematic statistics. Such statistics collect material for the problems which experience tells us are the most frequent and important in the history of States. In following these facts regularly the material (§§ 1 and 60) itself becomes of interest. The observation and discussion of a single object is not based entirely on the problem to be solved, but each object is examined to find the best possible and most fruitful investigations which might be undertaken in connection with it. It is necessary to study carefully the phenomena, to examine its causes and effects and their connection, and thus form a judgment as to the nature and practicability of the observations which seem to promise the most information.

Thus theories spring up as to how certain branches of statistics, as of population, industries, transportation, and the like, may be most effectively treated. These are questions of methods, but lead also to investigations of the nature of the conditions and changes of population, popular wealth and popular morals, public economy and national strength. This is the reason that a large number of influential statisticians hold that statistics is essentially the science of human societies (§ 56). In opposition to this view one may justly object that a systematic knowledge of human society which did not at the same time include the historical, geographical, anthropological, physiological, and, finally, also the philosophic consideration of it could not be, after all, a science. It can, none the less, be an important subject for instruction. This school of statisticians has produced, however, the practically and theoretically important branches of knowledge which we designate statistics of population, medicine, finance, economics, morals, and so forth. At the same time we should remember that we already possess a certain body of

facts for each of these subjects. We know certain regularities, probabilities, causes and quantitative relations, and principles of determining these relations.

If it should be asked how this conception and treatment of the material affect the features of the method described, it cannot be overlooked that, however exact and careful these statisticians have been, the methods of statistics require the same care and exactness for all problems.

THE TENDENCIES OF SYSTEMATIC STATISTICS.

Systematic statistics has given to statistical processes some peculiar features. Two things could not fail to affect the contents and mode of treatment. First, the observations are confined to one country and to essentially the same group of objects. Secondly, they are governed not by the special needs of a particular problem, but by general considerations which take up the problems in a general way, collecting materials for their solution, but not attempting a solution.

Confining the observations to one State, and making them continuous or periodical, render almost wholly superfluous one of the fundamental requirements, the determination of the aggregate and its limits in time and space. These are, then, matters of course. On the contrary, the possibility of future use gives the highest importance to the distinctions between the smaller subdivisions of the State. Even when the problem might be solved by the totals of the entire State a great number of subdivisions is deemed advisable. A large number of subdivisions are made for the enumerators, and while it is not meant that all these are distinguished in the results, it is true of the most important of them, counties, townships, etc.

Not only the aggregates, but also the units, are determined by general considerations instead of special needs. The group of enumerated units is extended in view of their probable causal connection as far as possible without over-

burdening the enumeration. The choice is made so as to avoid counting the units of a general nature, and also others which are merely subdivisions of them. The subdivisions are counted so that the sums will give the total. In this manner a more correct enumeration of the qualified units is secured, and at the same time the totals for a larger group of units obtained. These may possibly be important. Absolute uniformity of the enumeration in the time, determination of ideas and organization increases its value for comparison. There is frequently combined with the systematic preparation of the data the calculation of ratios for some of the more usual comparisons.

The entire system excels in clear distinctions between aggregate, unit, and ratio, and a plan of arrangement which furnishes easily understood results. This is also reflected in the presentation of the results. The fruit of all these labors are the great compilations of tables, in which the data are arranged with the greatest detail possible, but in which conclusions and definite problems are only exceptionally included. This form does not, however, warrant any conclusion that there is any difference in principles. All the results, as far as no special questions are to be solved, are to be classed as statistical material (§§ 73 and 78). It is not the final purpose of the investigations.

§ 91. THE PROBLEMS OF SYSTEMATIC STATISTICS.

Systematic statistics has been highly developed, and presents certain phases of statistical processes in a very favorable light. But it cannot be denied that it has done little to promote a correct conception of the real purpose of the process, and hence of the real character of statistics generally. Its influence may be characterized theoretically as tending to show that the investigation of the units, and not of the masses, was the object of statistical inquiry. This misconception permits of ready explanation. The aggregates have become fixed and appear to

be always the same. The group of things observed is apt to be always the same. It comprises the persons and things which have the greatest value for the people and the nation, and which, when investigated, give the best characterization of the latter. These persons and things are in their essential qualities typical, and are so treated in every statistical observation. All that is asked is, Do they exist? and if so, have they certain qualities? In the same manner they are observed in the changes of their mutual relations. And as the result of all observations there appears to be no other result than a certificate of these changes.

Hence it is not strange that attention should not be given to the aggregate, which is regarded as a matter of course, but that it should centre in the objects. It may appear to be more important and more useful to base any further investigation immediately on the persons and things. One may discover a gain for statistical processes in permitting an investigation of typical things with reference to their variable elements.

Unfortunately this apparently simple and attractive conception, however widely held, is inadmissible. For the correctness of statistical conclusions, as well as the application of results, it is confusing and in most cases dangerous. It forms a generalization for which absolutely no foundation is to be found in statistics and for which statistics cannot be responsible.

No statistical judgment deals with the unit, but strictly and only with the aggregate. The variable elements of persons and things otherwise typical, that are enumerated, are always counted in a specific aggregate and under certain specific circumstances. The qualities of the objects themselves, so far as they are not typical, or the subject of the investigation, are completely unknown. There is absolutely no presumption that these other qualities are typical or permanent in the aggregate observed, and much less beyond its limits.

When the person or thing has no relation to an aggregate it is wholly immaterial whether the variable elements are observed statistically or in individual cases. A generalization from them is inductive. If the quality is typical, statistical inquiry is unnecessary; if it is not typical, then it is only known where it is observed, or where a conclusion upon observations leads us to expect it upon the basis of probabilities. Even such a general rule that for 20 girls 21 boys are born, does not hold except within the range of the analogy, that is, within civilized States. Whether or not this condition exists in China nobody knows, and anybody who claims that it does places himself upon an anthropological induction, not upon a statistical probability.

§ 92. THE FORM OF THE QUESTION IN SYSTEMATIC STATISTICS.

Systematic statistics must have as a basis a definite aggregate in solving any problem, and its excellent and elaborate material occupies exactly the same position as any other. This may be shown again by an examination of the form of the question, which is the kernel of every problem.

The form of the question is, in the operations of systematic statistics, as a rule, peculiar. The questions raised in the preliminary observations to the compilations of tables, are not devoted, as a rule, to single aggregates. On the contrary, they discuss the aggregates having the phenomena in the greatest and least intensity and the relations between them in this respect.

It would seem that with this peculiar manner of discussing the material there must be some change in the nature of the problems arising from it. But closer consideration reveals the fact that the general theoretical scheme has not been departed from.

One question appears at a certain phase of every sta-

tistical problem. It occurs first in the comparison of analogies. It is the question discussed in § 87, of the variability of phenomena; *i.e.*, the question of the relations of the fluctuations. Upon this is based the determination of the average, the consideration of the minima and maxima, and the entire conception of the progression of the series. This shows that the discussion, when applied to the various aggregates, is only another form of treating the same subject.

Any attempt to answer questions upon the basis of this combination will show that it admits of no use for any purposes not derived from the analogy of the aggregates in question. Thus, in the example of § 75, the question is permissible, which district has had the greatest growth of population, or whether it has been greatest for the district A. But if the question is whether all the districts or the district A have had a large growth of population, these data are worthless, because the question calls for a different analogy. In this case each district separately or the entire number must be compared with these conditions at an earlier period.

But if we leave the field of quantitative judgments, where there is a greater measure of freedom, and proceed to the judgment of causalities, it is evident that for the indication of causes we cannot proceed from a plurality of aggregates, but only from the single aggregate, unless, indeed, the plurality is regarded as a whole. The question why in district A the growth of population has been greatest separates immediately the different districts. It must, therefore, follow that unless the results obtained are to be useful simply in accidental cases, they must form part of some previously arranged problem. It is characteristic of systematic statistics that it takes such problems into consideration. To argue correctly and avoid all errors we must go back to the starting-point required by the method; in other words, we must start from a single aggregate, or what is the same thing, a total of them.

§ 93. FORM OF JUDGMENTS OF CAUSALITY AND PROBABILITIES.

The form which the judgment of causality and probability is apt to assume, has also a tendency to prevent the proper conception of the methodical process. It follows from the fact that the labor of collecting the data and preparing the returns for publication is out of all proportion to that required to formulate the conclusions arising out of them. Yet these are the final results.

As a rule it is quite sufficient to prepare the material up to the quantitative comparison. This enables a specialist to form his conclusion in a short time. Hence a very large portion of the material is collected, and presented to the public in compilations, without any attempt at drawing conclusions. It does not seem advisable to do so until some necessity for it arises. When it is necessary to form a judgment one can seldom wait until some new special material for it has been gathered. By far the greater number of problems which arise must be answered from existing material.

The publications of official statistics have therefore the character of store-houses of information. The same may be said of private statistics which offer information for general use. Such are: Gothaische Genealogische Kalender (§ 18); G. Fr. Kolb, Handbuch der Vergleichende Statistik, 1857; Brachelli, Staaten Europas, 1853; Martin, Statesman's Year-book, 1864; Brachelli, Statististische Skizze, 1868; Legoyt, La France et l'Etranger, 1864; Michael G. Mulhall, The Progress of the World, 1880, and the Dictionary of Statistics, London, 1884.

When official statisticians discuss the conclusions to be drawn from data, they are apt to do so in periodicals and in historical or political works. In the bureaus themselves the discussion of such matters is limited to the needs of the organization.

Nevertheless, there are statistical problems requiring

very extended and complicated calculations and estimates for the conclusions to be drawn from them. But this does not place them outside of the usual processes. In this number should be classed, for instance, statistics of trade in commodities. The aggregate to be considered is the customs limits, and totals have no other divisions than those of time, where months or quarters may be distinguished. The units are the different articles, which are further distinguished as goods exported, imported, in transit, warehoused, etc. The material is counted by pieces and weight, the results examined and grouped. From this we can compare the different units in time and compare their quantity. A simple change in quantity does not show certain essential matters sufficiently, unless there is some grouping by degrees of quality, and the value of the total of each group determined.

This necessitates appraisement by experts, and for this purpose a comprehensive knowledge of prices is requisite with all the difficulties in its application which are pointed out in § 65. The statistics of trade are, therefore, worked over in a double way, and are one of the most important tasks of statistics (*Gesetz über die Statistik des Waarenverkehrs des deutschen Zollgebiets mit dem Auslande*, v. 20, 6. 1879; *nebst Ausführungsbest.*, 2 A. 1885; R. G. B., S. 261; *Central Bl.*, p. 676; *Stat. d. D. R.*, Bd. 43).

Another example, very different, yet very complicated, is to be found in mortality tables. The simplest rudimentary forms have been discussed (§ 14). More exact forms give rise to intricate and delicate problems (Appendix VII.). The simplest, as well as the most complicated, is a judgment of probabilities. The combination of the year of birth or the age of the dying with the living persons of a given population, furnishes the probability of death. If one goes a step further, and from a single death table or a comparison of several arrives at the order of death of a large group of populations, we find a statistical regularity. It is not necessary to discuss the similar problems, only

slightly less complicated, which might be drawn from the statistics of agriculture, of finance, of trade, or of medicine. The requirements of the technique show also the definite internal workings of the method.

D. THE REQUIREMENTS OF THE TECHNIQUE.

I. THE PROBLEM AND THE PLAN.

§ 94. THE SEPARATE PHASES OF THE TECHNICAL PROCESS.

There are certain distinct processes which are to be observed in the solution of every problem of empirical investigation, and upon closer analysis even in all our synthetical judgments. They proceed from the development of the idea, to procuring the means of proof, to the examination of the latter, to the answering of objections, and finally to incorporating the result in our general knowledge. In statistical problems these phases appear with unusual distinctness.

No matter what the statistical problem may be, it must proceed according to a *plan*. It is always a specific question which may be answered in several more or less accurate ways. The end in view and the resources which can be drawn upon will indicate in which manner and within which limits the answer is to be given. According to the choice made, it may be very simple or very complicated. But under all circumstances a definite plan providing for all the details is an absolute prerequisite. The means of proof are collected according to this plan. They may be gathered from material in existence or by special observations and enumerations of greater or less complication and extent. The means of proof must be subjected to a critical examination, they must be sifted, errors corrected, and compared with known facts. By this prepara-

tion they are ready for use. The question must now be answered and *presented* in its details, so that it is summarized, and yet sufficiently clear to enable an expert to examine the premises. Finally, provision must be made that the knowledge gained shall not be lost. It is necessary to preserve not only the conclusion, but, as far as possible, all that leads up to it, for future reference. It is probable that it will be needed to solve future problems.

§ 95. DEVELOPMENT OF THE PLAN.

A statistical problem grows out of a practical or scientific need. It is always a clear and distinct question as to some concrete relations. The answer to the question is always the number of times in which certain objects occur in a confused and changing aggregate, and, further, the relations of a quantitative, causal, or probable nature which such objects bear to one another. The answer can only be obtained by comparing the number of objects in aggregates of a like nature.

The range which remains open for the mode of answer is, as a glance at the method shows, very broad. The aggregate and the purpose of the problem are the only fixed elements. Within certain limits the choice of objects to be counted is free. Theory shows that one unit may sometimes be used in place of another (§ 81). This is especially true when it is wished to judge of abstract peculiarities or qualities of the aggregate rather than actual phenomena. In this case the objects counted act only as indications, which are to be chosen with reference to the reliance which can be placed upon them. And, further, the characteristics of the object chosen may be clearly distinguished and sharply drawn, or, on the contrary, may be less well defined, giving less exact results. Such characteristics might be chosen because of the facility of enumerating them.

The investigation can choose simple or complicated

methods in ascertaining the number. Instead of observation and enumeration, easy or difficult according to circumstances, estimates may be used. The use of estimates, particularly when they are based upon remote probabilities, is almost unlimited. It depends wholly on the degree of exactness required, or which can be considered satisfactory under the circumstances.

Finally, the choice of the analogous aggregates to be drawn into the comparison is (§ 74) as to extent wholly at the discretion of the person who is forming the judgment.

From all these considerations it follows that statistics is rarely at a loss for an answer to any question which may be put. Yet, in every case, with increased resources it can give a more satisfactory, more comprehensive and reliable answer. The most important question for the expert is whether the material at hand is sufficient for the answer, or whether new observations and enumerations are necessary and appropriate.

New investigations require for most statistical problems an extensive organization of officials. As a rule, it is not the statistician who first proposes the question and provides the means of answering it, but the State, corporations, economic or scientific bodies.

Hence the plan of investigation depends largely on the means at the disposal of the expert. These means will depend on the judgment of a superior official, who must be convinced of the amount of expenditure necessary to obtain a satisfactory solution of the problem.

The decision as to the mode of execution will depend upon the absolutely necessary requirements of the problem, the limit of time allowed, and the greatest pecuniary expenditure permissible. When this is determined, it may be considered whether the plan may be extended without interfering with the purpose or increasing the cost, so as to include further data of practical and scientific importance. In any case, the entire plan will depend largely from the clearness with which the estimates of the time

and cost for the different processes can be made and compared.

§ 96. ESTIMATES OF TIME AND COST.

Estimating the time and cost connected with statistical investigations is greatly facilitated by the fact that they consist of a repetition of exactly the same process for a large area. In a case of necessity it is practicable to make the enumeration for a small section in order to gain a basis of calculation.

The costs of every statistical undertaking are distributed over the preparation, the enumeration, and the examination of the results. The preparation includes complete organization, direction, and drill of the enumerators, the printing and distribution of the schedules for their use. The enumeration necessitates payments to the enumerators and other officials for their expenses and their labors. The examination brings with it the expenses for the services of more or less expert officials for the examination of returns, corrections, the tabulations, and in addition the cost of publication. Schedules and publications are estimated by the amount of paper and printing, labor by time, and, besides these, packing, expressage, rent, furniture, heating, lighting, and attendance, all at current prices.

It is easily seen, for an undertaking such as the enumeration of population, how many schedules are necessary for 1000 inhabitants, either for lists or for cards. The time for the enumeration, the number of questions asked, the amount of coöperation on the part of those counted, will determine the number which can be finished by an enumerator in a certain number of hours, and how much time will be required for reading for corrections.

The chief point is always the examination. If lists or cards have been collected for 45 million persons, and there are 12 answers for each person, it will be necessary to make and calculate 540 million items. It can by trial

be easily ascertained how many seconds are necessary for a small number of items, and for their addition. Five seconds for each item would make 2700 million seconds, or 750,000 hours, or 125,000 days of six working hours each, which finally would equal the labor of 417 men for one year or 139 men for three years. In a similar manner the most complicated estimates may be calculated. Every combination of questions is at least equal to a further question. All necessary reductions must be calculated by the number of multiplications and divisions necessary. Hence one can decide beforehand whether it is not desirable to omit certain items, because their consideration would involve too great an expenditure of time and labor.

In a similar fashion we may form from the details an accurate idea of the space which the printed tables will occupy. It is absolutely indispensable to form from the start a precise idea of the tables in which the chief results are to be grouped. The number of figures which will be required for each column and the number of columns are easily ascertained, and then the necessary breadth of the table. The number of lines depends altogether upon the number of subdivisions of the chief sum. The number of lines shows the length. Suppose the chief sum Prussia contains figures of 4 to 10 places in 50 columns, in all, for instance, 400 figures; and if this is to be divided according to the 54,000 boroughs, there will be required a breadth of 4 folio pages, and 1000×60 lines in length, *i.e.*, a work of 1000 sheets. The costs of composition and printing would come to about 100,000 marks, and an edition of 1000 copies would require 2050 reams of paper. In this way it is simple to calculate by figures and pages how far the subdivisions may be entered into when the funds at disposal are limited in amount. What cannot be printed is, as a rule, worthless, and should not be examined or collected.

(E. Engel, Die Kosten der Volkszählung Zeitschr. des K. Preuss. Statist. Bureaus, X. Jahrg. 1870, p. 42; Preuss.

Statistik, Heft XXIX., 2 Hälften Vorwort, p. VIII.; Erforderniss an Drucksachen, Preuss. Statistik, Heft XXXIX., 2 Hälften, p. 50; Kostenanschlag der Berufsstatistik von 1882, Drucksachen des Deutschen Reichstages, 5 Leglper, 1 Sess. 1881-82, N. 27.)

§ 97. FIRST DRAFT OF THE PLAN.

The decision as to the plan to be pursued in solving the problem may favor the use of material already in existence, supplemented by conclusions based on probabilities. It may be deemed advisable to furnish a basis for the conclusions by ascertaining certain data by means of the enquête or enumeration. All such methods are only means of facilitating the complete methodical processes which rest in principle upon enumeration. They are in part explained through the theory and technique of the process of enumeration. In using them one can never feel secure unless in a position to survey all the known data, and to examine whether it is based upon enumerations actually meeting all the requirements of the theory.

We must in theory have a plan, by which it is proposed to collect the means of proof. The plan when elaborated must provide clearly for the collection of the material, and also for the proper use of it in forming conclusions. For the technical execution of the plan, the utilization of the results is for the present of minor importance. The character of the data themselves will determine their use.

The first technical requirement is the composition of the regulations for obtaining the actual data, by enumeration or other methods. The enumeration is a practical undertaking, the first step in the realization of the plan. Once commenced, it will not admit of change or variation without endangering completely the value of the results. Hence it is absolutely essential that the plan should include a carefully worked-out scheme of the entire process of enumeration. As a rule, the execution of a plan is often

beyond the influence of the statistician who framed it. From it the enumerating official can only draw a few very general conclusions. Hence the statistician should never neglect to work out his plan into the last details, so that he himself could, if called upon, furnish all the regulations necessary. Only when this is done can there be any sufficient guaranty of the practicability of the plan.

The notion of the aggregate and of the units of enumeration must be so clearly defined that absolutely no room for misapprehension remains, and every step of the process must be carefully prepared in order to avoid confusion. There must be no doubt of the suitability and practicability of the organization. Appropriate provision must be made for the organs of enumeration, the means of observation, and the manner in which the observations are to be preserved and summarized. A schedule for the enumeration with the necessary explanations and directions must make unmistakably clear what things are to be counted, what particular characteristics, and what distinctions and combinations are to be noted. In the same manner it must be clearly understood exactly what result is expected. Hence no plan can dispense with a schedule for the tabulation showing at a glance all the elements which are to enter into the final result. It is only by comparison of these prospective totals with the questions upon the collection schedules that one can see how far the latter are complete. This comparison does much to make clear the nature of the whole operation.

None of these requirements can be neglected without danger to the plan. The complete contents and final purpose may indeed remain a secret of the expert. But for the public in general there must necessarily be definite regulations for the business of enumeration.

(Examples—*Statistique des Décès*, 1855, *Compte rendu général* (§ 48), p. 42; *Statistique du Système et des Institutions de Prevoyance*, 1855, p. 152; *Statistique des Finances*, 1857, p. 245; *Bericht, betr. Ermittelungen der*

landwirtschaftlichen Bodenbenutzung und Ernteerträge, 1871, Stat. des Deutsch. Reichs (§ 35), Bd. I. p. 102; Bericht, betr. die Statistik der See Schifffahrt, 1871, *Ibid.* p. 229; Plan für die Deutsche Forststatistik, 1874, *Ibid.* Bd. XIV. p. 52; Vorbereitung einer Deutschen Medizinalstatistik, 1874, *Ibid.* p. I. 156; Volks und Gewerbezählung am 1 December, 1875, *Ibid.* Bd. XIV. p. I. 24, and Bd. XX. Th. I. p. 1, 51; Preussische Statistik, Heft XXXIX., 2 Feb. 1882; Gesetz über die Erhebung einer Deutschen Berufsstatistik von 5 Juni, 1882, Stat. d. Deutsch. Reichs, Bd. LIX. p. I. 2, and Neue Folge, Bd. I. p. 4; Centralbl. d. Deutsch. Reichs, 1882, p. 48.)

§ 98. INSTRUCTIONS TO ENUMERATORS.

The instructions to the enumerators must indicate clearly the methods and scope of the investigation. It cannot be advantageous to have plan and instructions drawn up by different persons. If the same person composes both, one acts as a check upon the other, and compels the most unmistakable precision. If, on the other hand, a second person issues the instructions, it is clear that he cannot grasp so completely as its author the scope of the plan, and hence contradiction and confusion might ensue. The second person might not have the same feeling of responsibility for the imperfections of the work if the original plan were the work of another. (The instructions issued by the various German States for the carrying out of the census of industry of 1875: Stat. d. D. R., Bd. XXXIV., Th. I. S. 129 ff.)

The instructions require in every case the most pains-taking preparation and supervision. The process is here entirely bereft of any theoretical character which might be seen in the plan. The process acquires a concrete practical form. The logical character of the aggregates and units are not questions for the enumerators. The area is subdivided into smaller portions, provinces, governmental

districts, tax districts, court districts, city, county, or election districts, districts of registration, districts of chambers of commerce, army districts, and others sufficiently numerous. Provision must be made that the limits are not misunderstood or overstepped. In exceptional cases of common or disputed jurisdiction it may be necessary to come to some conclusion. (Stat. d. D. R., Bd. XIV., 1874, p. I. 24, § 9.)

The subdivision of time is the usual one of years, months, and weeks. When the condition at a given moment is to be ascertained, the point of time must be carefully noted.

Theoretically, a single instant is desired; practically, this is only approximately attainable. Hence it becomes necessary in an enumeration of population, for instance, to fix rules for distribution and collection of the schedules, for noting the seamen, travellers, the newly-born, and dying, and also for later corrections and additions. (Stat. d. D. R., Bd. I. 1873, p. 75, §§ 8, 10, 13, 16, 17.)

Just as little is there any conscious conception of the unit of enumeration as such. The things themselves are counted—persons, houses, and wares—they are distinguished by sex, conjugal conditions, age, religion, occupation, education; and as with persons, so also with houses and wares, cultivation and crops, phenomena of the sky, and, in fact, with every object of enumeration. It is, as a rule, the variable qualities of typical things which are concerned. Here are all the difficulties indicated in § 66. The idea which, in the plan, may be very clear, must be expressed as briefly and definitely as possible, yet so as to be commonly understood without difficulty. This is the labor of the expert, and cannot be relegated to another person. In particular, provision must be made for the manner in which distinctions of measure, size, weight, and value are to be made, and how these are to be applied to the things to be rendered capable of enumeration by such distinctions (§ 65). Further, it is essential to consider

how far an estimate of the aggregate may take the place of the actual measurement of such qualities, or whether estimate may be used at all in place of enumeration. Precise instructions are particularly requisite in dealing with things which are susceptible only of estimates and not of actual enumeration. (Stat. d. D. R., Bd. XLIII. Th. I. p. 57, and Centralbl. für d. Deutsch Reich., 1879, p. 855: Distinctions of wares, and tare percentages in statistics of trade. Bd. I., p. 102: Estimate of area and crops in agricultural statistics.)

All these things are best given as explanations and directions for the use of the enumeration schedule. The body which directs the enumeration immediately or indirectly cannot dispense with instructions covering the entire field, giving necessary provisions for the extent, organization, the initial steps, and time-limits of the process. When intermediate bodies summarize the results, in whole or in part, a schedule of tabulations is necessary.

The directions and instructions for officials and enumerators assume necessarily the character and structure of a law or regulation. (Examples, § 95.) They must prevent dubious or conflicting interpretation in all details, and form an objective standard for those who make and those who use the enumeration. Whoever undertakes to carry out the plan or to use the material gained must be careful not to depart from the instructions and the characterizations of the objects given therein. Criticism is directed to the interpretation of the contents, and we recognize very properly the ability of the statistical expert in his preparation of exhaustive unmistakable instructions. (Mittheilung der Instruktionen § 21 der Allgemeinen Bestimmungen in betr. der Volkszählung im D. R., Stat. d. D. R., Bd. I., p. 75; Neue Folge, Bd. I., p. 2, § 18.)

II. ENUMERATION.

§ 99. ORGANIZATION OF THE PROCESS.

The process is thoroughly concrete. It requires a fixed organization, observations, and the summarization and tabulation of the results. The phases of the process are different according as we make an observation of a fixed condition or a movement.

The distinction is to be seen at the outset in the organization of the more important undertakings. If the statistical bureau does not itself undertake the enumeration, it takes place through the agency of various organs of the general administration. Those which make the observations directly are often very various and often difficult to select.

The enumeration of a fixed condition, as of population, cattle, ships, etc., must arrange to investigate the entire field in a certain limit of time. The mobility of the objects requires a rapid—as nearly as possible an instantaneous observation. Close relations to less mobile objects (buildings, industries, farms) lead to the enumeration of the latter at the same time. Hence, such undertakings assume large proportions and require large numbers of observers. They are repeated periodically. In consequence they cannot furnish regular employment, and it cannot be avoided that persons without preparation and practice should be employed. Hence there is great necessity for some preparation and instruction. The plan has recently been adopted, where the degree of general intelligence would permit, that each house-owner or mill-owner, and, indeed, all heads of families should write their own answers upon a list or card containing the printed questions. The enumerator has in this case only a secondary function: he lends his assistance in case of doubt, and merely revises the results. (Stat. d. D. R., Bd. XIV. p.

I. 24, §§ 4-13, 18; Bd. XX., p. I., 72, §§ 7-9). The greater the division of labor, the simpler is the work of each individual. It may, therefore, be possible, as in Germany, to secure this labor from persons who, in consideration of the public utility of the work, do it without remuneration.

The chief reliance for the correctness of the data is in any case the good-will and interest of those concerned. Means to facilitate this correctness are good schedules, the coöperation and supervision of experts, rewards which may be forfeited by neglect, and penalties for refusals to make returns or for intentionally false returns. (Reichsgesetz betr. Berufsstatistik vom 13 Februar, 1882, § 5, R. G. Bl. 9.) Finally, we must include in many cases the self-interest of the participants as members of the community. (Preussische Gewerbesteuerrollen, Gesetz. vom 30 Mai, 1820, Ges. S. 133. Gutereinschätzungen der landwirtschaftlichen Kreditvereine).

Observations of movements requiring a much longer time must be made from one or more stations. They have a more complicated character, and are as a rule conducted by permanent and therefore expert observers. (Registration, Tax, Customs, Signal Service officials.)

The observation is greatly facilitated when the phenomenon appears periodically, or may be so observed, as in the case of meteorological and river-depth observations. It is necessary to arrange that, at every point when an occurrence of the phenomena is possible there should be an observer present, whose attention is called to the matter either at once or subsequently. These observers must have fixed districts in order to avoid omissions and duplications, and it must also be determined whether the double occurrence of the same thing is to be noted once or twice. (Statistik des Seeverkehrs, Stat. d. D. R., Bd. I., p. 240, § 5, 15.) In the first case it would be necessary to distinguish each individual by name or mark of some kind (§ 67).

In addition to proper organization and supervision, good

results may often be secured by the requirements of the business of administrative bureaus. Such are obligations which must be satisfied by money payments (taxes), registration, which, if neglected, entails fines or other losses (births, deaths, etc., or losses which are to be made good by insurance); finally, also, automatic registers, which require only periodical supervision (meteorological and hydrographic stations).

(Examples of very ingenious aids in the organization under the most difficult circumstances are to be found in the census and registration in East India: Report on the Census of British India, 1881, Vol. 3, 1883.)

§ 100. THE OBSERVATION IN ENUMERATIONS.

The observation is the decisive point of the process. It must permeate into the facts, find the occurrence of the phenomena, make the necessary distinctions, count without omission or duplication, so that a correct total may be obtained. The observation is based upon a direct and subjective act of perception. Theoretically, the statistical process consists of a succession of such acts with sharp and clear distinctions, but the technical practice assumes a very different form.

There is only a relatively small number of investigations in which the enumerator searches for the phenomenon and decides what shall be counted. Under ordinary circumstances it includes only the census of population and domestic animals, and these only in the most primitive form of direct observation. Nowadays, when the owners and heads of families make out the lists there is little for the enumerator to do. The observation is made by those who answer the questions, and is hardly liable to mistakes as to the facts, but only to misapprehension of the questions.

In many matters the same person makes the observation who makes the statistical tabulation. But these per-

Appendix IV. (*to* § 100). *Collection of Results from Enumeration Cards.*

Combination I. Population, Sex, Citizenship.

Combination I. Population, Sex, Citizenship.						Combination III. Sex, & Males Born in the										
Prussians		Other Germans		Foreigners		Total		Persons Personen	Total		Males Born in the					
M.	F.	M.	F.	M.	F.	M.	F.		Total	1875	1870	1865	1860	1855	1850	
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	
Totals I.	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	

Comb. II. Sex, Citizenship and Birthplace.

1. Born in the commune of the enumeration
 2. " " " Kreis " "
 3. " " province " "
 4. " another province
 5. " another German state
 6. " a foreign country

A musical staff consisting of ten horizontal lines and nine spaces. The first, fourth, and seventh spaces from the bottom are filled with black dots. The second, fifth, and eighth lines from the bottom are filled with white circles. The third, sixth, and ninth lines from the bottom are empty.

Totals L , same as L

From II.

Combination IV. *Sex, Age, Class, Conjugal Condition.*

1. Single
 2. Married
 3. Widowers and Widows
 4. Separated, Divorced

Totals IV same as III

Combination V. *Sex, Age, Class, Religion, School Education*

- | | | |
|----------------------------|--------------|--------------------------------|
| 1. Protestant | Total | { with School Education |
| | | { without " " |
| 2. Catholic | " | { with " " |
| | | { without " " |
| 3. Other Christians | " | { with " " |
| | | { without " " |
| 4. Jewish | " | { with " " |
| | | { without " " |
| 5. Other Religions | " | { with " " |
| | | { without " " |

From JV.

Totals V. { with " " " "
without " " " "

From V

Combination VII. *Sex, Occupation and Position in Occupation*

Independent		Assistants		Servants		Dependents		From VI.		Comb. VI. Scr, Age, Occupation.	
M.	F.	M.	F.	M.	F.	M.	F.	M.	F.	1. Agriculture and Forestry	
•	•	•	•	•	•	•	•	○	○	2. Manufactures and Mining	
•	•	•	•	•	•	•	•	○	○	3. Trade and Transportation	
•	•	•	•	•	•	•	•	○	○	4. Personal Service	
•	•	•	•	•	•	•	•	○	○	5. Professions, Officials, etc.	
•	•	•	•	•	•	•	•	○	○	6. No Occupation	
—	—	—	—	—	—	—	—	—	—	Totals VI. same as III.	

Comb. VI. *Sex, Age, Occupation*

- 1. Agriculture and Forestry
 - 2. Manufactures and Mining
 - 3. Trade and Transportation
 - 4. Personal Service
 - 5. Professions, Officials, etc.
 - 6. No Occupation

Totals VI same as III

sons, clergymen, registrars, assessors, receivers of taxes, meteorologists, make these observations independently of the statistical problems, and are led by very different motives to devote a higher degree of attention and care than would appear were statistical considerations the only reason for their labors. By far the greater part of the data, from the simplest to those of the automatic registers, which are useful for statistical purposes, do not belong in the first instance to statistical work. Statistics does not, as a rule, participate in the observation, but makes only a critical examination of the results. Hence statistical instructions rarely devote space to the act of observation. They are concerned more with the proper classification of objects already observed in order to increase their usefulness or statistical problems.

Certain phenomena (houses, horses, storms), which can readily be distinguished, may be simply counted by relying upon the memory, and the giving the total for the area or period. Objects of a more variable nature cannot be properly counted without more precise methods. This precision may be obtained simply by rubrics in which each unit when observed is noted, so that the sum of the notations will give the sum of the corresponding units. If we have a special column for female, for single, for married, or divorced persons, the number of dots or lines in each column will give the number of each class. It may be desired to sum up the distinctive qualifications not alone but in connection with each larger class of units (*e.g.*, divorced women). In this case a direct count is possible only for the larger class and not the special class. The observation of the distinctions would require another separation. Accordingly, there must be different forms of schedules (§ 68).

If qualifying characteristics are introduced in the schedules, and the number of each is found, we have a summarized view of the observation. The final summarization is not commenced. All these things are independent of

the source whence the data are gathered, whether from direct observation, from reports of the enumerated, or from the numberless lists required for the ordinary purposes of administration.

§ 101. THE OBSERVATION IN ESTIMATES.

When estimates and calculations are brought into use instead of enumerations, the observation assumes a totally different aspect. The abridgment of the statistical process under certain circumstances by the use of substitutes for enumeration has already been discussed. It is possible, and, in some cases, even unavoidable (§ 69). Theory always requires much the same thing, that is, the determination of the units in the aggregate. Technically the difference is very great if enumeration is employed or a substitute.

In one case the estimate attaches to a certain grouping in the condition of the units, which is supposed to be seen, so that in a certain sense the sums are observed and counted. Whether the groups are seen or remembered, in every case a portion of the operation of enumeration is dispensed with. Yet the object is considered directly. Indirect methods may be employed and more liberty allowed in making the estimate. The most common form is the conclusion of probability, already discussed (§§ 83, 86). The known results in analogous cases form the basis of the conclusion which judges the relation of the unit to the aggregate in the analogous aggregates to be identical with that of the aggregate to be investigated, even when nothing is known of the latter but the analogy. Such estimates are of universal application. No one hesitates, when conditions do not change, to estimate the taxes, crops, prisoners, railroad traffic, etc., of a country by the figures of the year before. Condorcet very properly declares that this is the calculation of the healthy human intellect. Nevertheless, he underestimates the im-

portance of complete conformity with the conditions of the method for absolute accuracy.

Estimates may also be made without a complete chain of analogies. In those cases where enumeration presents very great difficulties there are not apt to be any enumerations for analogous aggregates. Probability cannot be employed where the interest of the problem attaches to the variations within the same individual. Hence, estimates can be made on the testimony of experts.

The broad field of economic statistics is especially dependent upon such estimates. While it would not be impossible to determine by measurement, by enumeration, the area cultivated with the different agricultural products, it would be wholly impracticable. Estimates are usually required from local authorities, who may employ more or less complicated methods. But it is impossible to know by enumeration the harvest by grain and straw, roots, or hay. This would require for each large and small farm, data which the farmer himself could not furnish with any accuracy. For such purposes there is no other resource than the average estimates of experts. (Stat. d. D. R., Bd. I., S. 103; Motive, p. 116.) Similar conditions apply in the statistics of industry and trade. The product of manufacturers and the profits of tradesmen could be enumerated, for they are in the books. But like the tax office, statistics must be content with estimates. Even when the question is asked directly of the producers, their replies are largely estimates or supplemented by estimates (*e. g.*, Montanstatistik, Stat. d. D. R., Bd. I., p. 302, Bd. XXX., L. 12, 55; Bd. I., Neue Folge, p. 53). Consumption and production, wages, prices charged by mechanics, small traders, carriers, etc., are susceptible only of estimate.

The process is apparently most simple, for the expert is simply asked his opinion. But this opinion is the fruit of complicated processes. Unless he can base it upon enumeration, a direct survey of the groupings or analogies, he must resort to indirect methods. He may find some

support in the numerical relations of some other object which includes the object in question, or bears some causal relation to it, or is in some other way symptomatic of it. Experience or experiment will guide him in establishing a relation for the usual connection of the two objects, as, for instance, a relation of the harvest to the kinds of crop, or of the product in a spinnery to the spindles when the area of cultivation and the number of spindles is known. As the application to the absolute total is simple, the main question is often as to the relation. The expert knows by calculation that x distilled liquor tax indicates y mash mass. Hence, he infers that in a given district the distilleries use potatoes of z quality, with p exploitation. Hence, from the amount of tax, he can tell the consumption of potatoes for the given district and period. It is to be noted that his judgment holds good only for the given district and year.

Yet in this method there is always a certain definite course which the expert must pursue in his observation. He must give an average relation. He must have sufficient knowledge not only of one or of some cases, but of so many that he may assume, as in the case of probabilities, to have a just appreciation of the fluctuations between the maximum and minimum and of their approximate relation. According to circumstances this feeling may be justified as based upon causal technical or economic connections of a very definite nature, and thus the estimate be very accurate. On the contrary, accuracy in the estimate may require such a series of observations, that, like an enumeration, all the phenomena in question must be drawn into consideration singly, or at least in groups.

This shows the limits of the application of the processes of estimate and enquête (§ 69). It is impossible even from the most accurate observation of single cases, or of a portion of the field to be investigated, to form a judgment of the totality of the phenomena or of the entire field, unless it may be shown, and this ought, as a rule, to be proven,

that the totality of the phenomena and the entire field possess everywhere like conditions and relations.

§ 102. SUMMARIZING.

The result of the observation, by whatever method it may have been gained, is expressed in the number of units found in the aggregate. The summarization must include the sums and all the distinct subdivisions required by the instructions.

When, in simple cases, the observer simply gives the total for his district, or they are gathered from his lists, the summarization consists only in the complete total of all the districts.

In some cases the units cannot be added in the collection schedules, as they require separation for the distinctions. In this case a large schedule in shape of the tabulation schedule is prepared, and the data from the lists gathered under the proper titles by scoring for each case a vertical line with the customary horizontal line for the fifth or tenth case, so that the total can readily be obtained. If the distinctions are very numerous this process is very inconvenient and tedious. The paper is large, the proper column must be found, errors are apt to be frequent and the method therefore unreliable.

Since 1860, therefore, enumeration cards have come into use. They were originally introduced in Italy, and 1867 in Prussia (*Zeitschr. d. Preuss. Stat. Bureaus VII.*, 1867, p. 305) in the following form: For each unit in the collection schedule a card was prepared from the lists giving all the qualifications in the shortest possible form. A further step was to give such cards to the heads of families for them to fill out, one for each member of the household. This was introduced into the general German census of 1871 for Prussia, Lauenburg, Brunswick, and Hamburg, through the efforts of Engel. The simplification of the clerical work has led to its use in various fields (*Stat. d. D. R., Bd.*

II. p. 112; Nessmann, in *Zeitschr. d. Preus. Stat. Bureaus*, 1871, p. 271).

With the cards the distinctions are summarized simply by placing cards with like data in piles, and then counting the cards. In this placing and counting it is easy to attain great manual dexterity. Each pile can now be used for any number of other distinctions. The more numerous such combinations the greater must be the economy of time and cost which is gained by cards in preference to the older method.

(Appendix IV. shows seven comparatively simple combinations as to population. Including the ten columns omitted for women, it would require in an ordinary schedule 823 columns to be filled and counted. In these there would be for every 1000 persons 9000 entries. By the card method a single sorting and counting for each combination, *i. e.*, by sorting and counting 7000 cards the same result can be obtained more accurately and in one-third of the time. The explanations in the Appendix show how the cards are to be grouped and counted, after which counts they are to be thrown together again.)

As is shown by Appendix V., cards may be made which give not only the sums by counting them off, but which also give the sums in such a way that by placing one over the other they may be added up without further additional writing.

In all summarization, from the simplest to the most complex, the rule holds that no opportunity should be lost to prove the results obtained. The figures should be so arranged that sums of certain lines when added give the same results as the sums of certain columns, and thus all error avoided. (See Appendix IV.)

The grand total gives the final results as to the aggregate enumerated. It must always be looked upon as a desirable elaboration of the result when the aggregate is divided (§ 90) in the summarization into as many as possible appropriate portions. It has already been seen (§ 74)

that such portions have the value and character of smaller aggregates, only when in them exactly the same units have been observed as in the general aggregate. If there are data which do not unite to form aggregates, they are important only as accessory numbers for calculations.

Every complete summarization needs, according to the requirements of the theory, an explanatory statement as to the instructions, the mode of carrying them out, the effect of any deviation from the rules, and the presumptive degree of correctness of the result. This is necessary for the critique. The latter can be much more effective when the original data are given. (Preuss. Statistik, Heft XXXIX., 2 Februar, 1882, p. 65; Gutachten der K. Bezirksverwaltungsbehörden, Stat. d. D. R., Neue Folge, Bd. I. p. 53; Montanstatistik Bestimmungen, § 5 et seq.)

III. EXAMINATION OF RETURNS.

§ 103. EXAMINATION, CORRECTION, AND CRITICISM OF THE RESULT.

The examination of the returns is necessary in order to assimilate the matter to the plan, so that the conclusions necessary to answer the question may be drawn from it. It requires that the result of the new enumeration be critically examined and ascertained exactly, that the analogous aggregates suitable for comparison should be examined and arranged for that purpose, and finally that the results be reduced to ratios.

The basis of all further discussion is the degree of accuracy which may properly be ascribed to the results of the enumeration. It is indispensable therefore to determine from the standpoint of the most rigid practical statistician whether in reality the process has conformed to the plan and instructions, whether also the instructions issued have entirely agreed with the intentions of the plan; further,

whether the organization and capacity of enumerators and experts was adequate to a proper and reliable observation, and finally, whether, in counting up, no deficiencies and ambiguities have crept into the result. It is necessary, as far as the material allows, to test the clearness of each return and the mathematical accuracy of each calculation.

Besides such formal investigation there must be a continuous and attentive criticism of the figures, testing at every step their probability. A basis for such comparison may be found by calculation from typical relations. Within the results of the same enumeration a careful sifting of the matter will show deviating relations which were to be expected, and also concentration or scarcity for which there is no explanation. Finally, the comparison with the results for earlier years, or for analogous aggregates, even those only remotely related, leads us to be sceptical of particularly large differences for which no reason is apparent.

Such matters must be cleared up by subsequent research and the mistakes corrected. Any subsequent enumeration which may be necessary for such corrections must obviously take place with reference to the same period as the chief enumeration. (Stat. d. D. R., Bd. I., Neue Folge, p. 2, § 12.) Unquestionable errors can only be omitted; or, when no other means are at hand, be removed by proper explanations.

After careful application of all these sifting processes it remains to ascertain the totals which are to be considered the final results.

Errors are in any case unavoidable. It has often been proposed to express this in the numbers themselves by giving only round numbers. Without doubt the feeling of security is increased by using only the unquestionable figures, *e. g.*, the first three places of every number. This would indicate that possible mistakes were considered as less than one per cent. But even if such percentages

could always be given, the correctness of the number would obviously not be enhanced. Rounding-off the numbers always brings an increase or decrease, and the actual number seems preferable to this, and has greater probability.

Nevertheless, as already shown (§ 70), a judgment must be made as to the presumptive possibility of error in order to use the numbers at all for the present problem. How far this may be necessary for other problems should, unless the possibility of error is very small, be definitely explained and recorded for the future student of the material. Up to a certain point it is necessary to accord a certain amount of credence to the authority of the expert who prepares a statistical investigation. He should therefore be fully alive to the responsibilities of his position.

§ 104. THE COMPARISON WITH ANALOGIES.

When, possibly after considerable labor, the numbers have been obtained, no further progress has been made than the indefinite description pointed out in § 73. There is as yet no fixed method by which a judgment may be formed until a comparison with other and analogous aggregates has been made. The comparison is the same, no matter what kind of judgment the problem requires. Preceding paragraphs (§§ 82 and 86) have shown that all probability depends upon previously ascertained causality, and this upon quantity. Hence, whatever the ultimate end may be, the first step in drawing conclusions from the data is a judgment of quantitative relations.

Yet this first step is partially conditioned by the ultimate purpose. Every series of analogous masses in which like units have been counted will enable us to establish quantitative ratios which are unimpeachable in their form. They may have a value for certain judgments. For this reason caution is needed in using the large mass of the

systematic material described in § 90. While ratios and comparisons are frequently prepared for the aggregates and units of these comprehensive schemes, it should be remembered that a slight variation of the question (§ 91) often suffices to change the analogy completely and render the former comparison useless.

In a special problem there is always a special question which must be precisely stated and clearly comprehended. This shows where we must look for the materials for the comparison necessary in quantitative judgments. Unless complete agreement in the analogy of the masses is secured the agreement of the units will not fit them for comparison. Whether the analogies belong to a large number of simultaneously counted aggregates or are drawn from widely different periods or localities is of no consequence, either for the underlying idea and method of comparison or for its technical treatment.

If quantitative relations may be judged from such comparisons the second question of causality may perhaps be answered. It requires for its conclusions certain additional independent premises, for which the analogies must be found, examined, and compared, even though they may exist in the same aggregate. In similar fashion the judgment of probability or regularity requires a third technically distinct arrangement of the problem.

Each of these forms corresponds to a certain syllogism. They depend upon the enumeration of exactly the same units in entirely analogous aggregates and upon sufficient accuracy in the process. They all need critical examination of the results. This examination must go over all the points which would be important for a new enumeration, and must test particularly whether the method of enumeration or estimate, applied possibly with a totally different end in view, does not contain too large a margin of error for the problem in hand.

However large or small the group of analogies applicable to the comparison, it should always admit technically

of a systematic review, in which the aggregates and units, with their distinctions and their limitations, and with their totals, should clearly appear.

§ 105. REDUCTION TO RATIOS FOR COMPARISON.

Once decided that the aggregates and units are capable of comparison, the only further step is to reduce them to common terms (§ 76). For each comparison the series in which the maxima, minima, and fluctuations may be recognized and the average made, can only be derived from the same measure.

This reduction is a purely arithmetical operation, and it is entirely immaterial whether the final purpose may be a judgment of quantitative relations or a search for causality and probability among more complex conditions. But it is an essential factor, a connecting link between the material and the solution.

As a rule, all these reductions determine the number in which the units are found on an average in equal divisions of time or space, or how many units of one kind are found for a fixed number of another in the aggregates compared. The mode of calculation in the principal cases has already been indicated in § 76. Very complicated calculations may be necessary in these reductions. Theoretically all elements of the reduction, *e. g.*, the area of the aggregates, must be collected with the fundamental material. This is not always the case in fact, for it may be necessary to make a special inquiry for certain elements to make the reduction complete. Technically this is important, as it is not probably the concern of the usual organization (§ 99), but requires a special process to be united, as a rule, with the labors of the statistical organs.

Such special inquiries are all the more necessary, as it is not always easily foreseen what points of view may be most suitable for comparison in conclusions of causality

and probability. In some larger undertakings (production, trade, and similar subjects) series of units may occur, some of which are noted by weight, some by piece, some by measure, and which can be compared only when reduced to value or some other common term. Thus arises the necessity of the determination of price, value, or other matters, the possible intricacy of which was indicated in the allusion to the statistics of trade in § 93.

The similarity of the units may be so conceived, that though obtainable from the material, they may require very extensive calculations. An example is to be found in the death tables mentioned in § 93, when the problem is to compare the average life of the populations of different States and periods. A similar case is the comparison of different districts with reference to the medical aid within the reach of each inhabitant. (Stat. d. D. R., Bd. XXV., 1877, § 57.)

Even when these complicated methods do not enter into the question, the reduction of a large number of totals may require a multitude of multiplications and divisions, and involve the expenditure of a great deal of labor. Hence various technical devices are employed to facilitate the work.

Multiplication tables are useful when the same figures are used often as multipliers or divisors. For variable figures the Calculation Tables of A. L. Crelles (1858), "which save all multiplication and division of figures less than 1000," are of service; also tables of logarithms, with seven figures (v. Vega, Bremiker), or five figures (Bremiker, Albrecht, Schloemilch), or even four figures (Wittstein). Calculating machines render good service. Various constructions have been tried; the most satisfactory is the Arithmometer. It was invented by Thomas, of Strassburg, in 1820; produced at a moderate price in 1851 (240 mark), and has since 1880 been quite generally adopted in the more important bureaus.

IV. THE PRESENTATION OF THE RESULTS.

§ 106. REQUIREMENTS OF THE PRESENTATION.

Whoever has examined the material, and compared the analogous aggregates after their reduction to common terms, is in a position to give, if possible, a solution of the problem. The ratios show at once the quantitative relations for the aggregate investigated, what causalities may be considered as predominant, and in what number the phenomena are to be expected.

On the basis of these simple mathematical relations the statistician is frequently able to solve at a glance many statistical problems. This depends, of course, on the nature and accessibility of the material. No scientific propositions can be based on the conclusions of a single observer, however capable he may be. The specialist is often able to answer the questions from his general knowledge of the premises, and frequently he would hardly need the reduction to ratios. But if his labors are to be beyond question, and to have lasting value, he cannot omit to present his material and the whole process of proof in such a way that any sufficiently informed person might convince himself of the correctness and reliability of the conclusions, and find the solution himself.

This presentation is essentially a report on such portions of the process as may be necessary to establish the proof and furnish the means of testing it. The facts must be brought out as completely as possible, so arranged as to show their connection with the problem. The entire means of proof should be exhibited, and with them the calculations upon which it is based. The wealth of material even for the most ordinary problems would make demands almost impossible of fulfilment in meeting these requirements, were it not that the method of statistics furnishes peculiar simplifications whose importance seems to be scarcely adequately appreciated.

The strictly logical character of statistical investigation permits the presentation of statistics in an admirably clear and simple form. Statistical investigation deals with real facts, but by means of familiar ideas. On the basis of these ideas it measures, sifts, and systematizes the object investigated. Nothing not included in these ideas attracts attention, and just as the scheme groups the ideas with their distinctions, their species and genera, so the conclusions can also be similarly collected. Nothing in the ideas is changed. All that is new is the number of things corresponding to each found by observation. Hence a special form for statistical proof is possible which presents with the utmost precision and brevity the enormous number of varying qualities which have been ascertained. This is done by representing the logical interdependence of the ideas in the manner of presentation. This gives rise to tabulation and graphical representation.

§ 107. TABULATION.

The tabular presentation gives a complete account of the entire process by giving concisely the results of each step. The table gives briefly the ideas outlined in the plan. The tabular system is best applied when the horizontal lines are devoted to the aggregates, the vertical columns to the various units. The various aggregates are named at the beginning of the line, the units at the head of the column. When the meaning of the terms used to designate the units needs further explanation, a note containing extracts from the instructions serves the purpose best.

The numbers for units of the same sort are therefore placed one above the other. There can be no doubt of the meaning of the totals, and the eye detects differences readily in the figures where units, tens, hundreds are in the same vertical line. If reduced to ratios, the latter may be introduced into parallel columns of the same

scheme, and the comparison made without further effort. It is useful also to place in the table the area or size of the aggregates, and this occurs generally in the column immediately after the names. Logically also a fixed portion of the area is like an enumerated unit, and the total area a total of enumerated units.

All columns except those containing ratios furnish totals. These are the total number of the units of the same kind found in all the aggregates compared. Brought into a ratio with a total of the sizes, we have the average of the ratios by which the individual ratios may be measured. In the same way units which form part of a larger unit may be compared, and the percentual relations of their totals to the total of the larger unit determined.

All tabular presentations have similar contents. It is only a difference of form when the aggregates are in the columns, the units on the lines, but it is not so clear. The case is different when the aggregate is divided by position and size on the lines and by time in the columns. In this case the table can only include the figures for a single unit. In like manner the columns may be occupied by various units and the lines by their distinctions, or the reverse. In this case the table can only include a single aggregate, and the comparison is more difficult, as each aggregate needs a separate table. Such tables, however, show more clearly the influences of changes in the aggregates and the special relations of the same units.

§ 108. GRAPHICAL PRESENTATION.

Graphical presentation is a modification of the idea underlying the tables. It uses geometrical instead of numerical methods. It often shows the results in their geographical distribution instead of merely naming the sections as in a table. The object is to gain greater clearness and simplify the drawing of conclusions.

It is perfectly clear that, for every unit or 100 units, a

point, a line, or a small surface might be placed in the system of coördinates. (Appendix VI. *b. c. a.*) If the number is to be expressed by a line, it is, as a rule, measured upon a vertical line from a fixed horizontal line, and for the sake of greater clearness the points are united. One can distinguish subdivisions of a larger unit by different color or shading of the proportionate part of the line or surface.

Instead of drawing the lines perpendicular to a fixed line, they can be drawn from one point as radii and the surfaces may be colored. Surfaces may be indicated in squares, and shading or coloring be employed to indicate subdivisions. In each square a system of coördinates may be placed. But the further this goes the more the simplicity which is the prime object is lost. All such methods of indicating sizes are called diagrams.

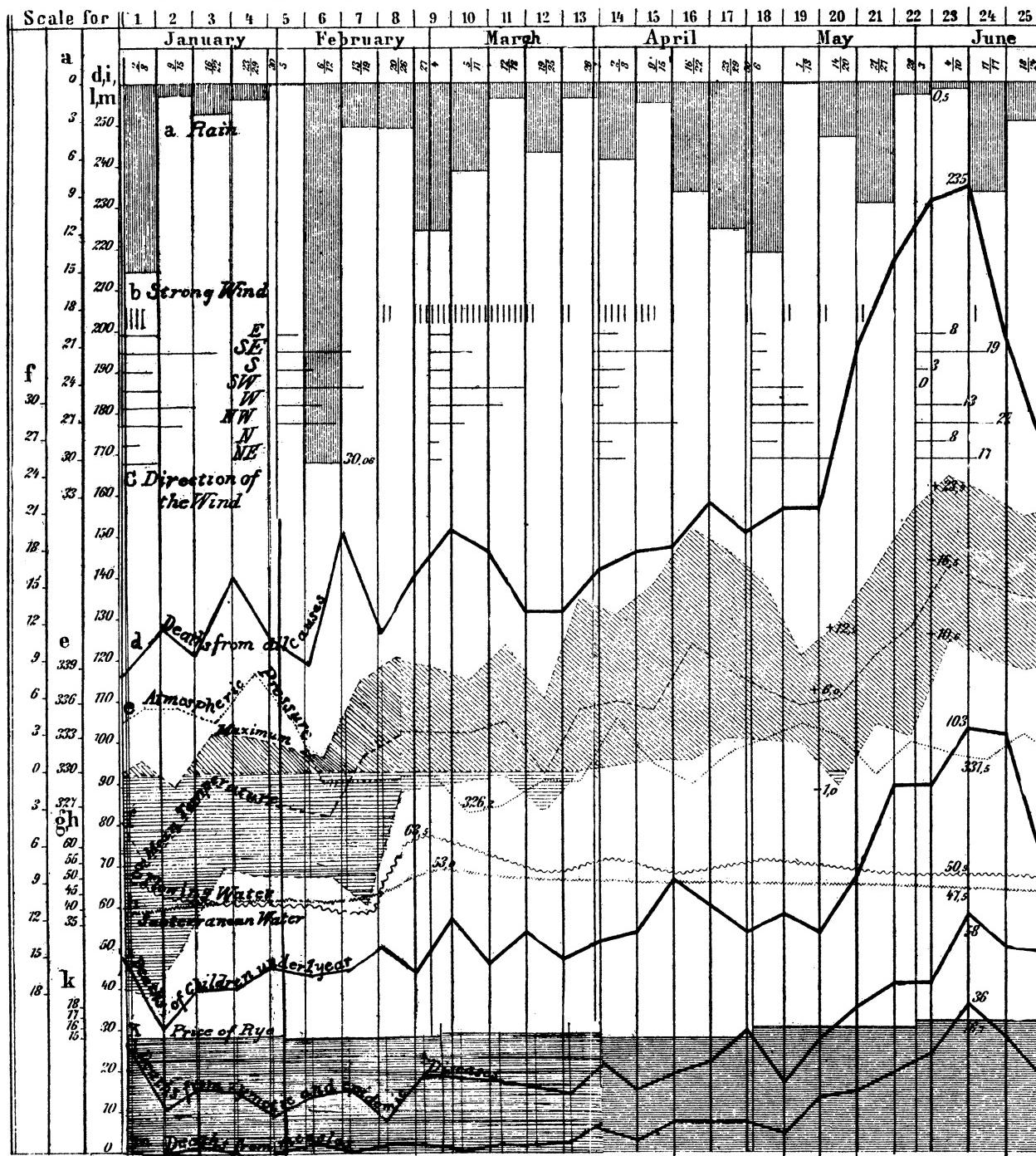
Hexagrams and chronograms are similar plans of showing the intensity of a phenomenon at various periods of time, by the rise and fall of a curve within a system of coördinates expressing a measure and time. If this occurs in an automatic register, observation and representation take place simultaneously. But the hexagram may also be constructed later.

A chorogram is the result when different aggregates are indicated by surfaces, showing the size of each, and portions of the surface are colored or shaded differently to indicate the result of the enumeration in each aggregate. It can be profitably employed to show the different degrees of the occurrence of a single phenomenon in different aggregates.

Cartograms are representations in which the geographical map shows the totals or the ratios of the different units by various inserted diagrams and hexagrams. The diagrams may here show a certain number of distinctions, but the limit is soon reached. (G. Mayr: Marey, § 57.)

In recent times solids have been used for these purposes. They have the advantage of three dimensions instead of

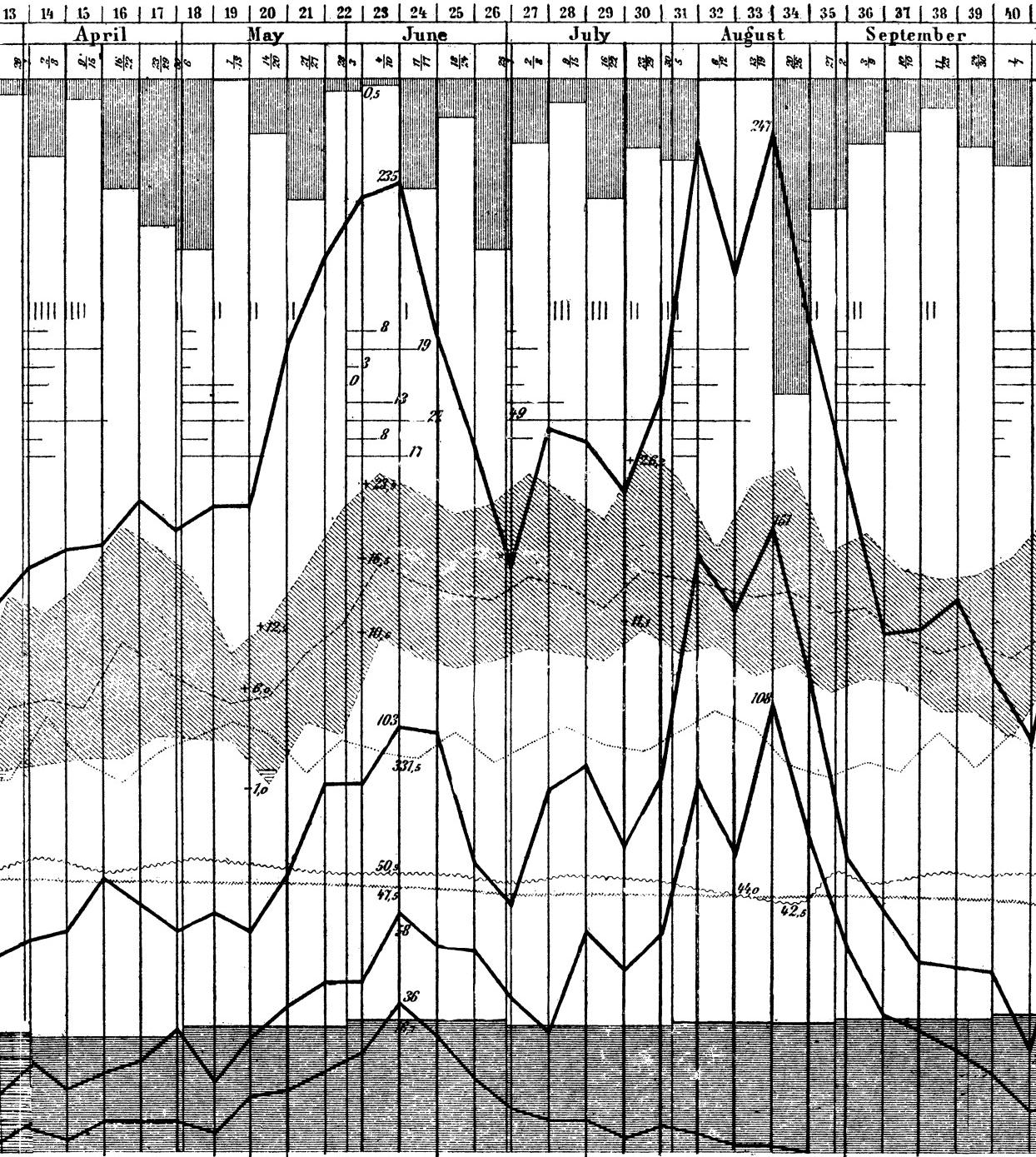
Appendix VI. (*to § 109, 110*). *Diagram used in Demonstration.*



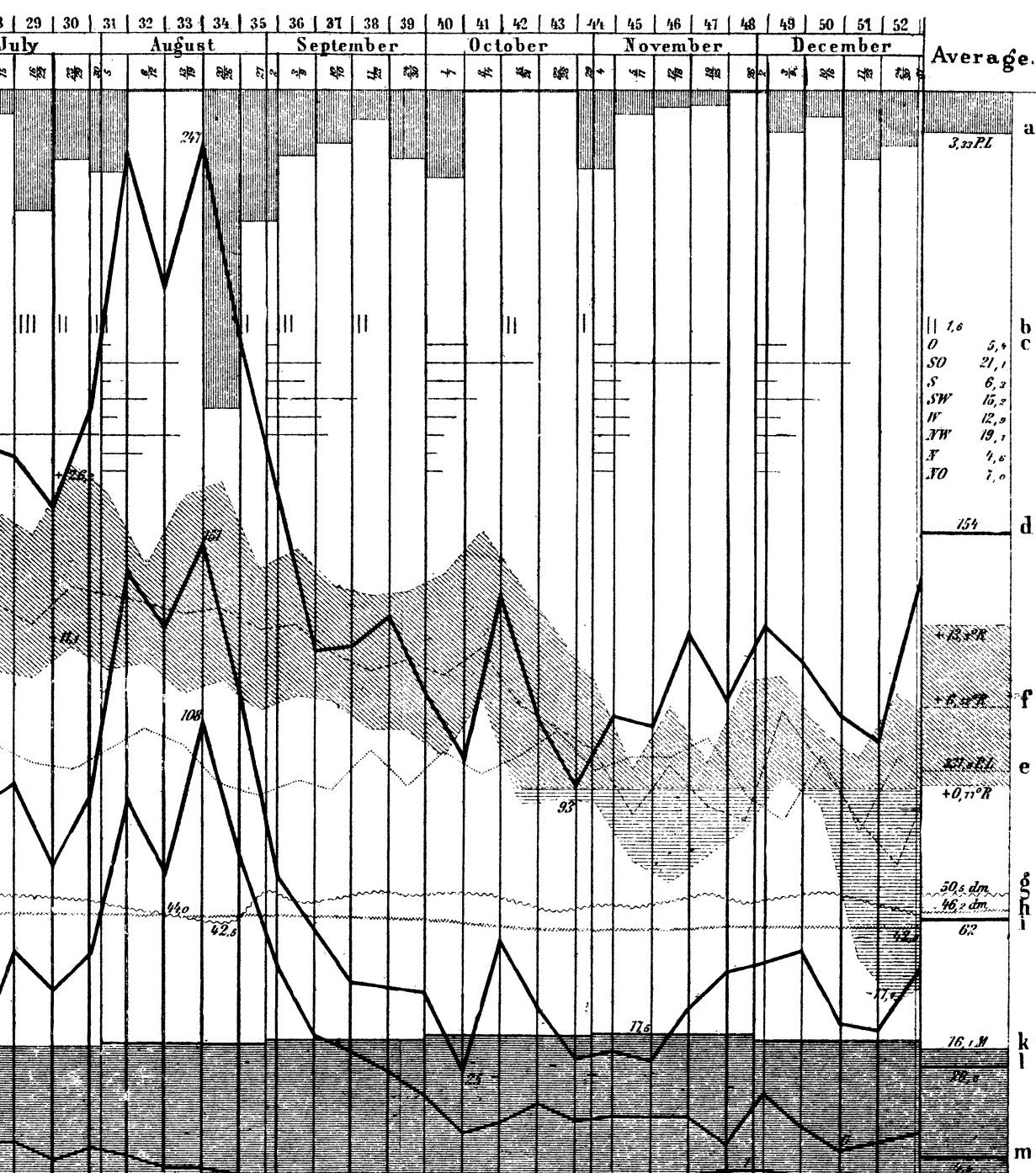
Based on the Statistics of Breslau, Series II., 1876—Scale : a, e, Paris Lines; b, The Winds of 2 and 4 degrees strength.

k, Mar.

ion.



, Paris Lines; b, The Winds of 2 and 4 degrees strength (over 2 m. in 1 second), observed 3 times daily; C, Observation number k, Marks per 100 kilograms.



n 1 second), observed 3 times daily; C, Observation number; f, Degrees Reaumur; g, h, Decimeters above average level; grams.

two. But the difficulty of constructing them, reproducing and preserving them, make an extended use of them impracticable. (Perozzo, § 57.)

§ 109. DEMONSTRATION OF THE QUANTITATIVE JUDGMENTS.

The demonstration follows almost mathematically from the presentation. It is based upon the certainty that an analogy exists between the aggregate which has been investigated and those with which it is compared, and that in all of them the things in question have been counted or estimated sufficiently accurately. This must be beyond a doubt, or else any further step is impossible. No one should omit to elucidate the means of examination by an explanation of the plan, of the instructions, the results, and the possibility of error.

Under the supposition that all requisites are complied with, the aggregates and the totals of enumerated things become merely quantities whose concrete character is not drawn into consideration. Conclusions are based on number and measure, on the mutual connections and relations of these comparable quantitative notions.

The regular statistical process advances from the descriptive material to a judgment of quantity, of causality, and probability. Practical demonstrations must follow the same course. In each special case it is a question at what point the problem in hand receives the solution desired. Circumstances may make the path longer or shorter, but its direction is always the same.

The demonstration finds a like expression in every example, but is perhaps clearest in the graphic presentation. For our explanation a case in which, as in Appendix VI., prominent extremes occur, will render the most efficient service. Suppose that for the investigation presented in Appendix VI. attention were called to the phenomena of the week ending June 10, 1876, in the city *B*, and that within the limits of time and space thus indicated, the

deaths be considered worthy of special observation. We find in the delineation of the problem in column 23 not only the unit d , deaths of all kinds, but a number of others which have been subject to observation and enumeration. Such are in the first instance parts of the unit d specially noted, as i deaths of children under one year of age, l deaths from zymotic and epidemic diseases (typhoid fevers, dysentery, diphtheria, measles, etc.), and finally as a subdivision of the last, m the deaths from measles alone. Other contemporaneous phenomena are also observed, evidently with the idea that they might have some connection with the deaths. Such are: a , the rainfall; b , intensity of the wind, and c , the direction of the wind; e , the atmospheric pressure; f , the atmospheric temperature; g , the level of flowing water; h , of subterranean water; and finally, k , the price of rye. That the winds, observed three times daily as to direction, and the price of rye are not observed specially for the week in column 23, but only for the month of June, is a defect. These data refer to a larger aggregate, the portion of which belonging to the aggregate 23 cannot be determined. Yet it must be assumed that any possible connection between the aggregate 23 and these factors is to be considered sufficiently characterized by the figures for the month.

It needs no further explanation to demonstrate that from the total d , observed for the aggregate 23, or from the sum of the units a to e , no one could know whether they were large or small. This question could only be answered by comparison with the aggregates 1 to 52. Here it is clear that it is a question of the place of the week June 4-10 in relation to the rest of the year. If one wished to compare the week June 4-10 with previous years in the same city or of other cities in respect to the units, the analogy would be quite different, and hence also the standard of comparison.

A survey of the unit d in the various aggregates to be compared shows at once that 23 has the maximum num-

ber. The final column shows the average for the year. By extending this average line across the page the fluctuations to each side would be apparent. The question of the quantity of the unit d in the aggregate 23 may be answered, therefore, by saying that it exceeds the average by nearly half; that like extremes occur only in the aggregates 31 to 34, while, as a rule, the fluctuations including those toward the minimum are much smaller.

In a similar manner the other units may be measured by reference to the standard given by the average for each, noting the deviation from the mean. Thus, it appears that in 23, besides d , the units a , b , f , i , l , and m occur in unusual strength, whereas the deviation from the average for the units c , e , g , h , and k is scarcely appreciable.

This is the basis of an exact, if not very far-reaching judgment as to the quantitative relation of the units. The treatment of the material could not differ essentially if the aggregate 23 represented a year of large exports, a district with large or small crops, or any other field of observation.

Whether the data for the various aggregates are the result of systematic statistical efforts collected for a special purpose, possibly even by means of estimate, would not, apart from the possibility of error, affect the mode of reasoning in the slightest. Nor would it be changed if instead of a single aggregate the entire field were the subject of the judgment (§ 90).

§ 110. THE DEMONSTRATION OF JUDGMENTS OF CAUSALITY AND PROBABILITY.

If the problem is not limited merely to quantity but includes also causality, the demonstration must consider the different sets of phenomena in turn. Suppose the question to be the cause of the extreme phenomena of the aggregate 23. One might at first look for k , the price

of rye, which might possibly indicate scarcity of food, and if so a probable cause of high death-rate. A close examination excludes this presumption entirely. The price of rye is subject to slight variations, and it is observed that in the aggregates compared the mortality is lowest when the price is highest.

The influence of epidemics is a possible cause. But the unit m does not prove it. It does show the maximum of deaths from measles, which may be considered epidemic. It shows the relation of this disease to all diseases of the class. The deaths, l , caused by epidemic and zymotic diseases, have, apart from the deaths from measles, a comparatively uniform and slight relation to the general mortality from the beginning of January to the middle of July, and again from the middle of September to the end of December. It is only for the aggregates 29 to 35 that an excessive increase of the zymotic and epidemic mortality is noticed, but it is observed that measles have no part in it. If, however, we consider the causality of the sum m for the sum d in the aggregate 23, it is plain that m , mortality from measles, gives no satisfactory explanation. Thirty-six deaths in m subtracted from 235 in d , leave still 47 in excess of the average 154, a difference almost equivalent to the extreme minimum (154-93).

Further explanation is necessary, and it may, perhaps, be found in the infant mortality. With an average of 62 it shows in 23 the large number of 102. But it is commonly known that measles are fatal chiefly to children. If we subtract the mortality from that source we find that the infant mortality has not appreciably increased.

Instead, therefore, of special influences, we must look to more general climatic relations. From the units g and h , which indicate the humidity of the soil, it appears that there is no reason to suppose miasmatic influences, which would have shown themselves in an increase of zymotic and epidemic diseases. The subterranean water at 47.8 dm. above low-water mark is higher than the average, 46.2.

This, with the flowing water, which at 59.9 dm. is higher than the average, 50.5, prevents the dangerous drying-up of the ground. The atmospheric pressure e is evidently indifferent, as it corresponds exactly to the average, 331, Parisian scale, or $29.397 +$ inches.¹ Observing the direction of the wind, it is noticeable that no southwest wind occurred, yet the others are all in about the average proportion. Of more weight is the fact that, as shown by b , it was less intense than in the other summer months. On the other hand it is not markedly below the average. The rainfall, a , of the week 23 is slight, only one-sixth of the average. But it can scarcely be looked upon as a cause of sickness, for though the preceding week had only one-third of the average, the one before that had had a precipitation of three times the average of 9.3 Parisian scale. Among all the elements prepared for this investigation there remains only the atmospheric temperature. This does, indeed, show very extreme conditions. The weekly average is the highest of all. The maximum of 23.9° R. is only once exceeded, in the 30th week, which shows 26.2° R., and the minimum of $+10.6^{\circ}$ R. is only a little less than the minimum of week 30. A peculiar feature is that the high temperature was developed very rapidly from a comparatively low one. In the middle of May the mean temperature was less than the yearly average—the minimum was -1° R. and the maximum only $+12.2^{\circ}$ R. Hence, the change was very sudden. The effect of this abrupt change seems to be indicated by the increase of measles.

This investigation would evidently be defective if the inductive or experimental knowledge which has been called in so largely to furnish premises could point out any other more probable general or special circumstance as the cause of the increase of mortality in the week in question. It would then have been necessary to measure these influ-

¹ In the Parisian scale $405.3425 = 36$ inches.

ences by enumeration either directly or by means of symptoms. If a critique does not indicate such circumstances, it must be admitted that the quantitative relations adduced in the presentation clearly show the rapidly increasing temperature to be the cause of the great mortality in the aggregate 23.

It is perfectly clear that the material which is prepared for such a proof of causality is capable of furnishing other similar proofs. The Appendix VI. shows in the extremes 31 and 34, already mentioned, another easily perceptible causality. A general view of the course of the lines shows that here the zymotic and epidemic diseases are the cause, and more precisely, among children under one year. In spite of the enormous increase of the mortality, the deaths of those of more advanced years show great regularity from the week 24 to the end of the year. All fluctuations in this part of the year are caused by the infant mortality, which is zymotic—epidemic, yet not attributable to measles. We must exclude as a cause of the fatality of disease in 31 to 34, apart from dearth (*k*), also atmospheric pressure (*e*) and wind (*b, c*). On the contrary, the subterranean water (*h*) shows an obviously unfavorable status. It has sunk from a level above the average to 2 dm. below, and at the same time the flowing water shows the lowest position, 2 dm. below the level of the subterranean water. Dangerous miasmas are highly probable. Nevertheless, the first extreme of high mortality precedes this sinking of the water-level. Yet the second extreme may have been partly due to this cause. Again, the chief cause appears to be the temperature, rendered less durable by long drought and feeble winds. The deaths are distinguishable from those of the week 23 in having a zymotic and epidemic character, but without measles.

Besides these causal connections the presentation illustrates in the 34th week another. It is in the obvious reason for the remarkably rapid decrease of zymotic and epidemic mortality, which with the infant mortality sinks

in the three weeks 34, 35 and 36, the greatest extreme, to a point below the average. The reason is the excessive rainfall of the week 34. The temperature does not seem to have been cooled off enough to account for the decrease. The explanation is the direct purification of the atmosphere and the closing of the pores of the earth by the great amount of rain.

If it is possible by such means to arrive not only at a negative result, that causes cannot be proved, but, in spite of the highly hypothetical character of the conclusions, to reach a satisfactory positive result, there is no need of any special demonstration that it is possible and desirable to pursue the matter further to conclusions of probability. Under like conditions for two aggregates, a cause proved to be operative in the one may properly be supposed to cause the like effect in the other, and its intensity may be measured by the intensity of the effect, compared with analogous aggregates. No one can doubt that an increased dry heat will increase the fatality of sickness, or that sharp and continuous rains will purify the air more effectually than weaker ones. According to the nature of the causal connection and the thing in question, the expression of such a relation in a definite number is more or less venturesome, yet if it is a question of using it statistically it must be done, always making ample reservation for the possible margin of error. With properly prepared material this is always possible. The entire presentation deals with definite quantities, and a relatively most probable quantity can, of course, be calculated from them. The causality in Appendix III. shows that with rye at 9.6 mk. a hectolitre, an average of 861 marriages per 100,000 inhabitants is the most probable for the four countries. By interpolation, according to the scale of absolute figures, the most probable number can be deduced for other rye prices, provided that with such defective material as the determination of price can (§ 65) furnish, they may seem to have value. To develop from probabilities the

regularities cannot be considered the concern of the single problem and its proof.

§ III. CHARACTER OF THE SOLUTION.

Presentation and proof furnish the answer which is the solution of the problem. The importance of the solution must not be overestimated. The peculiar processes of statistical methods fix very definite limits for the result. The problem requires an observation from a certain point of view of a variable and confused aggregate, definite only in its limits. Corresponding observations were made in similar aggregates. Whether the question concerns a single aggregate or a great number making up a total is wholly unimportant. It is only essential that the observation should be made with the use of the same logical premises.

These observations furnish the basis for all conclusions which have facilitated in any way the solution. Hence, it is clear that the latter is limited, as already seen (§ 91), to these fundamental premises. It rests strictly within the definite boundaries of time and space of the compared aggregates and of the characteristics of the things observed, forming the units of enumeration. This holds also in the field of probability. It cannot extend beyond the limits traced by the conditions. This limitation follows, as a matter of course, from §§ 82 and 86, for probabilities are only a species of proportion, where three known factors furnish a fourth. In like manner everything claiming to be a regularity is, as explained in § 86, always dependent on the analogy of the aggregates. Rules for the occurrence of qualities of things may be established inductively and experimentally with more or less precision, and the mutual relations of different things may be marked out. Such rules are derived from the general nature of the things, as shown by observation and experiment. Such a rule is not statistical. Statistics can only

be based upon the fact that a given combination of things has been observed under like conditions in a great number of aggregates, and hence under essentially unchanged conditions the same combination may be expected in analogous aggregates.

Where definite concrete things, such as cultivation and crops, trade, transportation, finance, tax statistics, and the like are concerned, this limitation is at once apparent. But in problems of a somewhat more abstract character the tendency to generalize is always dangerous.

For the validity of a rule the confirmation of numerous examples is more decisive, but the presumption of a regularity appears admissible from such problems and presentations as Appendix III. In this problem the price of rye is properly taken as a symptom of the cost of daily subsistence, and the rule is that cheap bread renders marriage easier. But whoever would conclude, for instance, that with 900 marriages per 100,000 inhabitants cheap subsistence, and with 700 dear subsistence, were to be inferred, would entirely overlook the fact that the question, whether the average 861 marriages per 100,000 inhabitants was many or little, was at the most answered only for the four states in question; that for other countries they furnish no standard. That the price of rye is not claimed to be a standard for the cost of living in remote lands hardly needs to be mentioned. Anyone who would conclude from the decrease of the price of rye, an increase for example of marriages among peasants, might be taught exactly the opposite by actual enumeration. The problem concerns only the total population, and does not indicate how the price of rye may affect single classes.

That no characteristics of the unit may be changed, that none may be neglected and none added, and that every more abstract or general conception includes such change, is more obvious. Much difficulty arises, for instance, from the use of the term city instead of city district, in enumerations of population and the like.

Just as little should the great importance of the solutions obtained be underestimated. The entire history of statistics is a steadily increasing testimony to its usefulness and indispensability. Its utility has been shown only within the limits in which alone it can be true. It is no fault that it cannot fulfil demands and expectations which have no foundation in those simple and logical premises which should be clear to everyone. Its methods and technique have always been aimed at the mark of reliable observations. Every empirical science seeks to increase its treasure, and each finds it, in spite of progress, poorer than desirable. Like the controversy of 1806 (§ 27), it would only be an illusion to see in this limitation of statistics an abandonment of ideal aims. The demands of that day were simply that the judgment of statistical aggregates might not be extended to combinations of phantasy until it stood firmly on the ground of measured facts.

V. PRESERVATION AND APPLICATION OF THE RESULTS.

§ 112. COLLECTION OF THE RICHEST POSSIBLE MATERIALS.

The last requisite of statistical technique is the proper preservation of the results attained in the treatment of statistical problems. The solution of every scientific problem is of permanent value. It does not disappear with the immediate occasion for it, and so statistics must furnish lasting achievements for the further development of systematic knowledge. This is all the more indispensable in statistics, as its technique requires so frequently the use of earlier results, which cannot be obtained by subsequent inquiries.

This demand may mean preserving the original material; *i.e.*, the original lists, schedules of arrangement, and collation, with the necessary explanations (§ 68), and in many cases this must suffice. But the purpose can only

be satisfactorily obtained by printing. Only by such multiplication of the results obtained is a proper use of the material possible, a comprehensible and handy form attained, and the occasional whole or partial destruction with which manuscripts are always threatened averted. The presentation (§§ 106, 107) takes into consideration the printing with reference to the contents and form. With the publication the work becomes public property.

Just as with the collection of the material, so with its preservation, systematic provision has very fruitful results. The useful results of one's own investigations, as well as those of other persons and other periods, are collected, and placed in readiness for possible use. So far as practicable, scholars and scientific and private institutions should attempt this for the material in which they are interested. But it is the province of the State, which in the nature of things has far the greater number of statistical needs and functions, to provide also for the preservation of statistical material. There are some general maxims to be considered for the organization of these official collections of materials.

§ 113. THE ARCHIVES OF OFFICIAL BUREAUS.

Each State endeavors to build up a system of official statistics, as explained in § 88, which shall be appropriate to its own conditions, as extensive and as logically connected as possible. At the same time the official bureau must endeavor not to lose any statistical results which are carried through by correct methods and are capable of critical estimate. This must extend to private statistics and the more important publications of foreign States.

It is necessary in the first place that the appropriate material which may exist at home or abroad should be known to the bureau. The greater part of the statistical publications does not come into the book trade, or excites so little attention that it is not noticed in advertisements and

catalogues. The reason is the very general custom of exchanges between the various institutions, which has the effect that few copies are sold. It would be wholly undesirable to change this. But attention must be given first to such publications as are not exchanged, and second to the regularity and extension of the exchanges. Instructions to the various public organs to send in their publications may be necessary. At the same time the purchase of works of reference and compendiums upon all branches of economics and technology, technical dictionaries and lexica of foreign languages, also copies of statutes and regulations, and particularly of the best and most complete topographic charts, is indispensable. This material increases from year to year in bulk, and the older material has lasting value for statistics. Provision must always be made for an appropriate space for the reception and classification of the material and its storage.

Not only the collection of the material, but provision for its easy accessibility, is very important. Catalogues should include not only the title but also the main points of the contents, which are often very much mixed and dissimilar in the same work; also the more important essays, and so forth, that they may be easily found. (Preussisch. Statist. Bureau, Katalog, see § 50; Zeitschrift, Jahrgang XX., 1880. Beilage.)

The less the official statistician is, as a rule, in a position to draw conclusions from the statistical matter on account of the necessary work of preparation, correction, and publication, the more he must desire to see such scientific treatment accomplished by private persons and scholars. Yet such investigations need not be expected, and are for the greater part impossible if the archives are merely open to investigators, but like ordinary libraries presuppose a knowledge of the works in those who look for them. They should rather be arranged like historical archives. There should be well-informed persons who are, if desired, willing to point out the material, and when necessary to ex-

plain it. (v. Scheel, Organisation der amtlichen Statistik, 1869).

§ 114. APPLIED STATISTICS, POLITICAL ARITHMETIC.

The limits of exact data and of the conclusions which may be drawn from statistical problems have been shown in § 111. It may, however, appear that the limits of the statistical field have been drawn too closely, if certain combinations commonly designated as applied statistics or political arithmetic are not drawn into consideration. These are based essentially upon the union of numerical results and other empirical maxims, with strictly statistical data. It would, however, be hopeless to look here for any peculiar form of knowledge. Inductive and experimental knowledge as well as deductions come into play. From the interweaving of such typical relations and comprehensive abstractions with statistical results, rich and fertile fields of inquiry may arise.

What is understood by political arithmetic founded by Petty (§ 13) is chiefly an extension of statistical enumerations and probabilities by calculations based upon technically necessary or typical relations. When the number and variety of the ships of a nation have been statistically ascertained, any seaman can easily tell how large must be the crews and also the seafaring population of the nation. The technical expert can tell easily the consumption of raw material, the necessary machines, the approximate production, the circulating capital, and other essential conditions of the enterprise from the number of spindles. Obviously all these data come by calculation from the empirical knowledge of the person who makes them and from the premise that the conditions estimated have necessary and typical relations.

The combinations can, on the other hand, be of a more abstract nature. Say said, that without the aid of statistics political economy would scarcely be an empirical science.

This is not correct, as Say meant it, that if statistics gives the causes or effects of its facts, it becomes political economy. Yet political economy has two sources whence its principles are drawn. Either the typical characteristics of a number of private economies which have been observed are applied to the nation, a course which, on account of the differences of economic and ethical qualities of individuals, is ambiguous and unreliable, or it gains them by the enumeration or estimate of direct or symptomatic phenomena within the entire nation or of so much as is concerned, *i. e.*, statistically. The latter can alone give an exact basis for further abstractions. In the same way geography, history, and other sciences include facts statistically ascertained and abstractions from these facts.

The scientific conception of statistics is of the utmost importance in this union of inductive and statistical conclusions. If it is considered the science of a certain object one must either include all objects participating in such combinations in the limits of statistics, or else draw a line when the consideration of an object ceases to be statistical and belongs to another science, and the statistical data utilized are to be looked upon as subsidiary to the latter.

If we consider statistics from a standpoint of a method which examines its objects from a point of view closed to other methods, then the single problem of applied statistics becomes simply a portion of empirical knowledge investigated more or less by statistical processes.

CONCLUSION.

§ 115. THE CHARACTER OF STATISTICAL THEORY.

It has been demonstrated that ever since the beginning of history extensive statistical investigations were requisites of political and social life, and how in consequence an-

tiquity and the middle ages show in various fields of practice some empirical skill. Scientific statistics is also the immediate outgrowth of the practical needs of State and Church in modern times. They existed for a long time as a mass of unconnected and partly heterogeneous matter, until it came within the mighty revolution of ideas, which since the beginning of our century has permeated and clarified all scientific knowledge. The idea of unity in the entire field of scientific statistical activity brought with it also the consciousness of a peculiar statistical method. The tremendous development of statistics in public administration, as well as in many departments of economics and other sciences, went hand in hand with a clearer perception of correct and exact processes. In spite of the most divergent views as to the scientific position of statistics, there exists entire unanimity among statisticians as to the demands and conditions of the statistical method.

Our presentation of the subject has aimed to explain this method in its logical foundation and in its technical application, in order thus to gain the principles of an unambiguous and complete theory of statistics. The result of this treatment may be briefly summarized. Theoretical statistics is the doctrine of a strictly systematic process of investigation which can be properly applied to every concrete object conceived as complex and changeable. Statistics is capable of furnishing knowledge attainable in no other way as to the connections and relations of things in this changing aggregate. This doctrine constitutes a well ordered system of ideas, demands, and principles which have definite relations to the general theory of perception.

Whatever position may therefore be accorded to statistics in a system of the sciences, it cannot be denied that unless it is considered, with Sigwart (§ 58), as a part or branch of logic, it must be classed properly with logic and mathematics, or at least, as is done by Rümelin (§ 57) and Haushofer, with the methodical sciences of critique and hermeneutics.

A peculiarity of the method is the fact that its comprehension is not based upon a particular portion of previous knowledge. Although for many problems it may be necessary to have very exact knowledge of political and legal matters, or of technical purposes and conditions, kinds and denominations of means, tools, and materials employed, and also of the decisive relation of the events, it is not in all these cases any special technical knowledge that is required, but simply just such ideas and points of view as are not inaccessible to a man of general education and common sense.

But just from this fact very appreciable difficulties arise. Because statistics operates almost exclusively with the ideas of everyday life and combinations of numbers, which are very simple, it makes very extraordinary demands upon the specialist. It is unfavorable in the first place that the purpose of the problem is abstract, that it is not connected with the direct operations of daily life. It certainly is very interesting to compose a plan, to propose problems, or to draw or substantiate conclusions—but these operations are rare exceptions. As a rule, one is concerned simply with the treatment of the material, collation, sifting, and correction of the results, continued for years. One list looks like another; there are the same ideas, the same sets of figures through hundreds of sheets, always the same common-place things and everyday distinctions. At the same time we must not overlook any possible misapprehension of the idea of the unit of enumeration, whatever form it may assume, nor any improbability in the data which may include errors. Assistants may indeed be charged with the discovery of false or changing designations of things, of errors of calculation, of writing, etc. But all more serious doubts are the concern of the interest, attention, and continued patience of the specialist properly so-called. Hence it is obvious that any proposed lightening of these labors involves not a merely personal question, but possibly the value of the result.

On the other hand, because it deals in ideas plain to everybody and in figures which are easily comprehended, statistics is, more than other sciences, subjected to misleading and trivial use and criticism. Because of its simple elements, its conclusion and data always assume a form of expression apparently comprehensible. Interpreted solely by one's general knowledge, changed in its form and repeated in this new shape, it can only be a case of luck that the careful choice and limitation of the ideas still is applicable. Yet the entire system of conclusions is based upon this fact. Often when doubt arises as to the correctness of such traditions, the means of corroboration are lacking, and even when these are known nothing is easier than falling into new errors by small mistakes which so easily occur, unless there is a certain consciousness of the requirements of criticism and the mode in which the data are presented. Hence arises that peculiar contradiction in the opinions held as to the value of statistics, that one doubts completely the value of the figures, and another expects from statistics what is unattainable.

The means of decreasing these unfavorable conditions are worthy, finally, of our attention.

§ 116. THE WORK OF THE PROFESSIONAL STATISTICIAN.

That statistics in its methods makes great demands upon the professional statistician is due partly to the necessity of completely comprehending the purpose of practical problems arising in the most varied fields of political and economic life and scientific research, in order to make them susceptible to statistical treatment. Everything to be considered must either directly or by symptoms assume the form of things capable of enumeration, possibly of such as have qualities which can be measured. The indefinite ideas and conjectures from examples, which fill our ordinary thought, can never take the place of the complete observation with its numerical results. To determine what

aggregates may be considered analogous and compared, and whether their enumerations are of value, requires sharp discrimination and often comprehensive knowledge and research. Causality and probability cannot always be ascertained without a profound penetration into actual and historical premises and conditions.

But the real difficulties are not to be found here. On the contrary, these demands when they arise only stimulate and interest. The chief requirement is to keep up the interest, the continued tension, which enables the statistician to handle each of these endless repetitions of the same ideas and totals with the same energy. There are no means of attaining this except the change of the demands and the increase of responsibility for their fulfilment. The tedium of monotony and the consciousness of not making personally the finishing touches, are the greatest enemies of good achievements and capacity.

It is therefore highly desirable to make the specialist as far as possible head of the statistics for a certain district, so that he, himself, with not more than one scientific assistant, can attend to all special matters which cannot be considered merely mechanical, and hence all the problems engage his attention in comparatively rapid variation, and lead him to the desire and necessity of carrying on these works originally, intensively, and definitely.

The organization of bureaus for smaller districts is to be preferred to the establishment of a single centralized institution for other reasons. From the centre of a small State or a province or a district there remains a possibility of that local and personal knowledge which is of the utmost importance for estimating the value of different organs and their mode of procedure, for detecting errors, and for ascertaining the reliability of the conclusions drawn. The examination of the returns is also essentially facilitated. They are not in such an enormous number as to escape the supervision of the director, and they do not come from places and conditions so distant and different

that mistakes cannot be much more easily detected, and corrections much more easily undertaken with promise of better results.

Such a small bureau can give assistance and instructions within its district in a way impossible to a large central institution. The local officials can apply directly to it for explanations, which it would be difficult to obtain from the general publications. The small bureau can also apply itself to problems of local importance which a central bureau could not properly supervise, though it should undertake them. (v. Scheel (§ 113), *Zeitschrift f. d. gesammt. Staatswissenschaft*, 1869, 1 Heft.)

The cost of statistics is in no sense greater for decentralized than centralized organization, since the same labor is to be accomplished. At the introduction of the modern card system it was supposed that it would be cheaper in large factories. But this was only until its conditions had been learned by trial. When all the steps and all the accessories of the work are fully known, when uniform instructions and schedules have been distributed to the smaller bureaus, we must consider their less expensive office rent and salaries, the immediate supervision of the director, as well as the greater demands which a centralized organization makes upon the general administration.

There must of course be a central institution superior to the local bureaus. It provides the uniform plan and for the uniformity and collation of results. In § 50 attention has already been called to the benefits which have resulted from the position of the German Statistical office as erected by the resolutions of the Bundesrath, as compared with the numerous statistical bureaus of the single States, chiefly in promoting the method. But it would not do to conclude from the recognized superiority of the growing labors of this bureau peculiar advantages in centralization. This would overlook, on the one hand, that the German Empire was so fortunate as to possess in the first director of statistics a man of extraordinary critical endurance and acute-

ness, and of remarkable clearheadedness in the conception of the technical requirements of the work ; and it would overlook, upon the other hand, that the most admirable achievements of a centralized institution cannot make up for the loss of those advantages which the lack of smaller bureaus necessarily entails.

§ 117. THE POSITION OF STATISTICS IN GENERAL CULTURE.

The means of promoting the proper appreciation and use of statistics, outside of statistical bureaus, must, as a matter of course, consist in spreading correct knowledge, and in the increase of the number of those who have been instructed in theoretical statistics.

In this connection it is highly desirable to have a certain number of young administrative officials pass as assistants through the statistical bureaus. This would involve considerable inconvenience for the director, for he would have to instruct them, and it would require some time before they could furnish sufficiently useful and reliable work. Their occupation could not therefore be calculated for a very short period, but should be placed at from four to five years. It could, however, be counted to their credit, like any other administrative work, for they would receive great benefit from it, and take with them a knowledge and experience and a critical insight into statements of fact, which would be of service in any office to which they might be assigned, and promote generally the proper comprehension of statistics.

Such an arrangement would aid in discovering the persons, who are by no means numerous, especially qualified for statistical work, and thus variety in the labors of the directors of statistical institutions could be more easily introduced.

For such preparation statistical seminaries have been created. One was established by a Ministerial Rescript of August 18, 1862 (Preuss. Minist. Blatt. f. d. innere Verw.,

1862, p. 257), in connection with the Prussian Statistical Bureau. Other seminaries have been established by professors of political science at universities, such as Halle, Leipzig, and Strassburg. It is important whether such institutions are conducted with the intention of schooling statistical specialists or for the instruction of students. The first requires, besides the usual scientific preparation, extensive practice from occupation with the first material to the conception of definite problems. It is presumably to be attained best when it is looked upon as a profession, and gained by work as a clerk and then as assistant in a bureau.

For the purpose of instructing students of political economy and others who are not looking forward to a statistical career, the seminar cannot attempt extensive undertakings which would occupy the student for some time, and thus draw his attention from his studies. Such seminars are limited to smaller essays, usually to discussion of examples and demonstrations. Nevertheless it is just as fruitful for theoretical education as it is indispensable. It differs chiefly from the statistical exercises which form the practical part of the lectures only by certain appliances, such as a regularly accessible library, working-rooms, collection of schedules, diagrams, means for calculation and drawing, and the like, which are very desirable.

Yet the question of the preparation of specialists is by no means the all-absorbing topic. The demand for them is not so great but that a substitute may be found. This will be all the more easy if the effort to bring into the general culture a clearer conception and comprehension of statistics, which stands obviously in the foreground, is successful.

In this effort the burden of the task falls upon the universities. It is important to spread as far as possible the theory of statistics, for theoretical education does not mean anything else than gaining knowledge from systematically arranged principles and illustrations, in con-

trast to the experience gained by a practice of the art. For it is evident that the numerical results of statistics without reference to the fields of knowledge which they serve—geography, anthropology, political economy, history, etc.—cannot be subjects of instruction except as examples of method and of its possible results. The method itself can, however, be presented as a connected body of thought.

It is indeed a body of doctrine preëminently adapted as preparation and support of a general scientific education. It is closely united to the first bases of human perception and logical thought, and, unlike theoretical logic, does not develop from these ideas an apparently abstract and abstruse system of syllogisms, but shows directly and clearly how these logical connections are turned to account in problems of economic and political life, of a thoroughly practical and indispensable character. In every new explanation it shows in a new form how the slightest error of an idea or smallest logical mistake can render large undertakings useless, and in their consequences misleading and even dangerous for the judgment of important facts of political and social existence.

The theory of statistics shows also the value and application of logic, and must heighten the interest in critical thought. It leads to a comprehension of the earnest and firm purpose of statistical endeavor, to a consciousness that requires strict truth and love of the truth, that the proper treatment of the conclusions or data, which will not permit or excuse any self-deception, involves a serious responsibility, that may properly be designated as a demand of the public conscience.

For the youth of all professions, particularly those of official position, the theory of statistics is a very appropriate field of study. When its way is opened more and more among students we may expect a reaction upon political bodies, the press, public opinion, and general culture. We shall then become accustomed to a more

critical treatment of statistical questions; ambiguities of conception, of interpretation, and proposals will decrease; statistics will be more exactly applied in their true fields, and, with proofs of their value, the general estimate of them will be higher.

NOTE. Appendices III., IV., and VI. are placed opposite the paragraphs to which they refer, in order to facilitate comparison with the text.

APPENDIX.

APPENDIX I. (TO § 14) HALLEY'S TABLES OF MORTALITY.

Halley's Report—An Estimate of the Degrees of the Mortality of Mankind, Drawn from Curious Tables of the Births and Funerals of the City of Breslau, with an Attempt to Ascertain the Price of Annuities upon Lives (Philosophical Transactions, Vol. XVII., for the year 1693, 15 pp. in No. 196, and 3 pp. in No. 198)—says that in the tables under consideration for the five years 1687 to 1691, the age and sex of all who died in the city of Breslau is registered with all possible care and exactness, and that the figures are given by months and compared with the births. Complete materials for the year 1691 were also preserved. It would appear, however, that Halley's observations are based mainly on the following table, which, as the main result of Neumann's work, had been transmitted to him with the other papers. The upper line gives the age, and the one just below the number of persons of that age who died in each year. Where no number is given in the upper line the lower figure indicates the number of persons who died each year in the period between the previous and succeeding ages:

The two points in brackets are not to be found in the essay, but this must be a printer's error, for, as Knapp has pointed out, they must be supplied. (*Theorie des Bevölkerungswechsels*, 1874, p. 125.)

Halley has, as shown in his calculations, supplemented this table with other observations, so as to secure a regular progression, which might serve as a general scale, as follows:

7	.	9	.	14	.	18	.	21	.	27	.	28	.	35
12	10	9	6 $\frac{1}{2}$	6	6	6	6	6	6 $\frac{1}{2}$	7	7	8 $\frac{1}{2}$	9	
36	.	42	.	45	.	49	.	54	.	56	.	63	.	
9	9	10	10	10	10	11	11	10	10	10	10	10	10	
70	.	72	.	77	.	81	.	84	.	84	.	84	.	
II	II	II	10 $\frac{1}{2}$	10	8	6	4	2	1 $\frac{1}{2}$					

The principal change is in the number of deaths between the 9th and 18th years. He increased this, as he held the small number in the table to be merely accidental. He was correct in this, but went too high on his part.

Halley says that it appears from the Breslau tables that in the years in question there were 6198 births and 5896 deaths (1173 $\frac{1}{2}$ yearly, therefore). Hence the growth of the population is about 64 in each year, or about one-twentieth, but this is probably equalized by recruiting for the Imperial military service. But as this is problematical, the number of births, however, well known, it may be supposed that the population of Breslau grows by 1238 births annually. It appears further, from the same tables, that on an average 348 die in the first year of their age; that only 890 reach the age of 1 year; that between the completion of the 1st and that of the 6th year, 198 die on the average, and hence only 692 survive the sixth year.

On this basis he constructed the following table. This, he says, shows the number of the population of Breslau in all ages from birth to old age, and shows, therefore, the probability of death at each age. This shows how annuities may be calculated, which until then had only an imaginary basis, and also shows what are the chances that a person of one age should reach some more advanced age. In his language, "this table does show the number of persons at the age current annexed thereto, as follows:"

Age current.	Persons.										
1	1000	8	680	15	628	22	586	29	539	36	481
2	855	9	670	16	622	23	579	30	531	37	472
3	798	10	661	17	616	24	573	31	523	38	463
4	760	11	653	18	610	25	567	32	515	39	454
5	732	12	646	19	604	26	560	33	507	40	445
6	710	13	640	20	598	27	553	34	499	41	436
7	692	14	634	21	592	28	546	35	490	42	427

43	417	50	346	57	272	64	202	71	131	78	58
44	407	51	335	58	262	65	192	72	120	79	49
45	397	52	324	59	252	66	182	73	109	80	41
46	387	53	313	60	242	67	172	74	98	81	34
47	377	54	302	61	232	68	162	75	88	82	28
48	367	55	292	62	222	69	152	76	78	83	23
49	357	56	282	63	212	70	142	77	68	84	20

Age.	Persons.	Age.	Persons.
1	5,547	63	1,694
14	4,584	70	1,204
21	4,270	77	692
28	3,964	84	253
35	3,604	100	107
42	3,178	Total	34,000
49	2,709		
56	2,194		

The first series shows how many of 1238 persons born in Breslau die in each age class. The second series shows a general order of mortality, as Halley called it, based on the former.

If to explain the two series we assume that the 1000 persons noted by Halley in the age current 1 represent the births of the 1st year, then the entire calculation must be reduced to 1000 instead of 1238, and this reduction must appear in the further figures. But, since for 1238 persons born, 890 are said to survive the 1st and 692 the 6th year, the number at the age current 2 must be not 855, but 719, and hence all the following figures must be too high. Hence the population of Breslau could not be counted, as is done above in the second series, by adding together the first. If, on the other hand, it should be supposed that the number of persons opposite each age current represented the average number of persons living in each year, it would be seen that 1000 instead of 1238 was not appropriate in view of the greater mortality of the early months. Hence it would be seen that, though the following numbers might be averages, it would not agree

with Halley's statement, that 692 survive the 6th year, to find the same figures in the middle of the 7th year. All subsequent figures would, therefore, disagree.

If the figures opposite each age current are not those living at the beginning of the year or at the middle of it, they can only represent those who survive each age. This agrees with the supposition that of 1238 persons born 692 survive the 6th year to find this figure opposite the age current 7. Hence, working backwards, we find surviving the 5th year 710; the 4th, 732; the 3d, 760; the 2d, 798, and the 1st 855. On the other hand, Halley gives this last at 890, and the number 1000 could only be the average number during the first year.

Although Halley laid no great weight on these calculations of estimates, we have still no ground to suppose that he made any mistakes in his figuring, and the difficulties in his methods cannot be solved with such material as has been preserved.

(G. F. Knapp: *Theorie des Bevölkerungswechsels*, 1874.

J. Graetzer: Edmund Halley und Caspar Neumann, Breslau, 1883.

E. Rehnisch: Review of the last work in *Göttinger Gelehrten Anzeigen*, 1883, p. 1576.)

APPENDIX II. (TO § 48) SYSTEM OF RESOLUTIONS OF THE INTERNATIONAL STATISTICAL CONGRESS. (Compte-rendu général, St. Petersburg, 1872.)

A. THEORETICAL AND GENERAL STATISTICS.

1. *Theory and Technique of Statistics.*

- General treatment of methods.
- Graphical methods and cartography.
- Uniformity of terminology.
- Statistical instruction.

2. *Statistical Organization.*

- General principles of organization; central commissions; scope of statistical organs.
- Statistical publications.
- Exchange of the same.

3. *Organization of the Congress.*

- Resolutions on organization, statutes, and work of later Congresses (temporary regulations).
- Plan for comparative international statistics.

B. PRACTICAL OR SPECIAL STATISTICS.

1. *Territory and Nature of the Country.*

- Cartography in relation to catasters and the transfer of property.
- Hydrography.
- Application of natural sciences to statistics.
- Meteorology.

2. *Place of Residence.*

- Statistics of large cities.

3. *Population.*

- Census: data to be collected, methods of enumeration.
- Subsidiary questions to enumeration of population, necessary for a general view of the nation.
- Population registers.
- Movement of the population.
- Death tables.
- Emigration.
- Basis of ethnographic statistics.

4. *Health and Sickness.*

- Statistics of geographical influences on health.
- Statistics of deaths.
- Statistics of epidemics.

Statistics of mental diseases.

Statistics of institutions for the sick.

Military medical statistics.

Comparative statistics of health and mortality of the civil and military population.

Statistics of accidents in transportation, factories, mines, and furnaces.

5. *Real Property.*

General resolutions, division, and extent of property in land.

Modes of ownership.

Transfer of ownership, prices, and indebtedness.

Catasters.

Buildings.

6. *Agriculture and Grazing.*

Agricultural investigations, data to be collected, and methods to be pursued.

7. *Fisheries.*

Statistics of fisheries.

8. *Mines and Furnaces.*

General resolutions.

9. *Industry.*

General statistics of labor.

Special statistics of industries.

10. *Production and Consumption.*

General resolutions.

11. *Laboring Classes, Prices, Wages.*

General resolutions.

Budgets of the laboring classes.

Prices and wages.

12. *Public Charity.*

Resolutions on statistics of the systems and arrangements for public charity.

13. *Money, Weights, and Measures.*

Uniformity of money, weights, and measures.

14. *Commerce.*

General resolutions.

Foreign commerce.

15. *Transportation, Shipping.*

Statistics of country roads.

Statistics of railroads.

Statistics of inland navigation.

Statistics of maritime navigation.

Statistics of postal service.

Statistics of telegraphs.

16. *Banks and Institutions of Credit.*
Statistics of stock corporations.
Statistics of banks.
Land credit.
Circulation of bank notes and paper money.
17. *Insurance.*
Resolutions on the statistics of insurance.
18. *Charity and Care of the Poor.*
Statistics of the poor.
Dependent persons.
19. *Public Education, Art, and Science.*
Schools and public education.
Institutions for instruction in the fine arts.
Institutions for the preservation of scientific collections.
Institutions for the preservation of collections of the fine arts.
20. *Justice.*
General resolutions, judiciary organization.
Statistics of criminal courts.
Statistics of civil and commercial causes.
21. *Prisons and Police.*
Statistics of prisons.
22. *Army and Navy.*
General statistics of land and naval forces.
Special statistics of marines.
Medical statistics of army and navy.
23. *Finance.*
General statistics of finance.
Yearly income of the nation.
Statistics of expenditure.
Finances of cities, the church, corporations.
24. *Statistics of Municipalities.*
General resolutions.
25. *Statistics of Colonies.*
General resolutions.

APPENDIX V. (TO §100) ENUMERATION CARD FOR ADDITIONS.

	<i>a</i>	<i>b</i>
Conservatives.	Imperial Party.	National Liberal.
	Progressist Liberal.	Clerical.
	Progressist.	Poles.
		Social Democrats.
		People's Party.
		Particularists.
		Protest Party.
		Undecided and scattering.

In the first line above: Number of votes for each party; in the second line, votes of each party in per cent. of all valid votes cast. (2)
See the Remarks (8)

Name of the Election District

Geographical Position: (8) North Germany; East, West, Middle; South Germany.

Urban or Rural Population: (3) Purely urban, including a large city, without a large city.

Prominent Religion: (3) Chiefly Protestant: more than 75 per cent of the population Protestant; less than 75 per cent, Protestant.

Chiefly Catholic: more than 75 per cent. of the population Catholic; less than 75 per cent. Catholic.

The electors form the following per cent of the nomination

The total votes cast form the following per cent. of the total number of electors

The votes cast for the Deputy, or Plurality candidate, form the following per cent. of the total number of valid votes cast.

The votes cast for the Deputy form the following per cent. of the total number of Electors

(See the Remarks on other side.)

REMARKS.

1. For each election district in which the candidate was elected in the first election, a white card is to be used. For first elections which were not decisive, a yellow card is to be used. For the decisive after elections, a blue card is to be used.

After elections which are not decisive are not considered.

The party of the successful candidate, or the plurality candidate, is to be underlined with red, that of the principal opposing candidate with blue. In addition, this must be written out as required by the blank.

2. The votes are to be counted by parties, whether given to one or to several candidates.
 3. For the answers to the questions as to the geographical position, urban or rural population, as well as prevailing religion, a special list is given, from which they can be obtained.
-

This card is not filled out in the election district, but in the statistical bureau from the election returns. The contents between the columns *b* and *c* can be treated like the ordinary card to obtain results by sorting as shown in Appendix IV.

In the columns *a* and *c*, on the other hand, totals are entered by units, tens, hundreds, etc., in exactly the same way on each card. They can be placed, therefore, one over the other and these totals added without further transcribing.

The column *b* contains percentage calculations, but if it contained totals they could be used for addition, after the column *a* was completed, by cutting off or folding over this column.

APPENDIX VII. (§ 93) THE CALCULATION OF MORTALITY TABLES.

The calculation of a table of mortality or survival is the solution of the problem of finding how many persons, in a given district and period, who were born in the same year will survive in each succeeding year. The proof may be based on the gradual diminution by death of an actual or ideal generation.

An actual generation would include all persons born in the given district, in the given period of one or several years, from the time of birth until the expiration of the last person. In an imaginary generation, on the other hand, the total number of persons living and dying in the district for the period in question are so united that the proportion of persons of each age who die to the persons alive at that age, gives the picture of the life and death of an actual generation.

The first method dates from Herrmann (§ 44), and has also been attempted in Baden, Prussia, and Cisleithania. It has been shown to be impracticable, on account of the need of maintaining the observation 100 years and the changes in the population through emigration. The second method is generally in use and satisfies the main purpose of the question; that is, to find the present mortality at different ages. Its proper conception and application is very complicated. It is rendered especially difficult by the fact that it is necessary to compare the living population, as ascertained on a given day, with the deaths for a whole year, and yet preserve the equality in the ages. The discussion of this condition of the problem is found as early as Wargentin. It has probably been stated by none so clearly and instructively as by K. Becker.

K. Becker has discussed the theory in detail. (*Anhang zum 1. Theil des IXten Heftes der Statistischen Nachrichten über das Grossherzogthum Oldenburg.*) But the shortest and simplest statement is to be found in a rather obscure place (*Stat. d. D. R., Vol. XX. p. 145, Bericht über die Reichs-Medizinalstatistik, vom 7. Oktober, 1874.*). Here he says:

"To find a sufficient expression for mortality, we must be able to ascertain it for distinct age periods, and, if we would be in any way exact, for each year of life. This is the necessary basis for any further consideration of mortality. As a matter of course we must know the ages of the dying. But this alone is not sufficient. On the contrary, it is essential to know the year of birth of those who die. This is shown by the following illustration:

"Suppose we are to find the mortality in the first year; that is, show how many of a number of persons born die before completing their first year. These data are given:

Born in 1861	7218
Deaths of persons from 0-1 year of age in 1861 . . .	936
Deaths of persons from 0-1 year of age in 1862 . . .	848

"How shall we make our calculations? We cannot compare the births of the year 1861 with the deaths of persons from 0-1 year of age in 1861, since the latter were manifestly not all born in 1861, but partly in 1860. Just as little may we properly compare the births of 1861 with the deaths, as given above, for 1862, since the latter include children who were born in 1862. To solve the problem of finding how many of the children born in 1861 died in their first year we must take our recourse to a hypothesis, either that the deaths of the year 1861 or of 1862 all come from the births of 1861, or that of the deaths of 1861 one-half were from births of 1860 and one-half from those of 1861, and in the same way that half of the deaths of 1862 were from 1861 and half from 1862—mere hypotheses that may be in glaring contradiction with the facts.

"The proof is quite as incomplete if the persons dying are separated not by age, but by the year of birth, as when we have the following data :

Births of the year 1861	7218
Deaths of persons born in 1861:	
Died in 1861	647
Died in 1862	387

"Here we know, indeed, that the last-named persons were born in 1861, but the 647 who died in the year 1861 form only a portion of the persons born in 1861 who died in their first year. The remainder are to be found among the 387 who died in the year 1862, some of whom, however, must have been within their second year. Again, we must have recourse to a doubtful hypothesis.

"A second illustration may be taken from a more advanced period of life. Suppose we are to ascertain the mortality in the 63d year (*i. e.*, of persons from 62 to 63 years of age), in other words, to find how many persons who reached the age of 62 died before they became 63 years of age. We have the following data :

Population at the close of the year 1861, at the age of 62 to 63 years, or, what is the same thing, population born in 1799 living at the close of the year 1861 . .	1598 (A.)
--	-----------

In the first case the deaths by ages.

Deaths at the age of 62 to 63 years:

Died in 1861	73 (B.)
Died in 1862	71

In the second case deaths by year of birth:

Deaths of persons born in 1799:

Died in 1861	59 (C.)
Died in 1862	84

"Neither in the first or second case can we ascertain the number of persons who became 62 years of age and the number of those who died before the age of 63. The persons who became 62 years of age in the course of 1861 consist of:

"(a) Those at the age of 62 to 63 years at the close of 1861, or those born in 1799 (See A. above).

"(b) Those at the age of 62 to 63 years who died during 1861, and who became 62 years of age in 1861 (not in 1860), or, in other words, who were born in 1799. This number cannot be found from the figures given above, since the number (B.) includes persons born in 1798, and (C.) persons who died at the age of 61 to 62 years.

"The persons who became 62 years of age in 1861, and died before reaching 63 years, consist of the persons of 62 to 63 years of age who died in both years 1861 and 1862, and who were born in 1799. This number cannot be found from the data given any more than that marked (b) above.

"Therefore, we must again, as in the first illustration, have recourse to hypotheses that are very unreliable, and cannot promise safe results.

" This can be avoided only when we have the facts not only for the age of those who die, or the year of birth, but for both of these. Then the calculations may be made in the following fashion:

FIRST ILLUSTRATION.

Births of the year 1861	7218
Died at the age of 0-1 year:								
In 1861 { Born in 1860	286
{ Born in 1861	647
In 1862 { Born in 1861	239
{ Born in 1862	609

"Accordingly, of 7218 persons born in 1861, $647 + 239 = 886$ died in the first year. The mortality is, therefore, $\frac{886}{7218} \cdot 100 = 12.27$ per cent., or the probability of death in the first year = 0.1227.

SECOND ILLUSTRATION.

Population at the close of the year 1861 at the age of 62-63 years	1598
Died at the age of 62-63 years :	
In 1861 { Born in 1798	33
{ Born in 1799	40
In 1862 { Born in 1799	32
{ Born in 1800	39

"Accordingly, in the course of the year 1861, $1598 + 40 = 1638$ persons became 62 years of age. Of these $40 + 32 = 72$ died before reaching 63 years of age. The mortality is, therefore, $\frac{72}{1638} \cdot 100 = 4.39$ per cent., or the probability for 62-year-old persons of death in the 63d year, 0.0439."

* * * * *

In this way the probability of death (w) is calculated for persons at each age, and correspondingly the probability of life ($1-w$).

Let us assume that the probability of death has been calculated in this way on the basis of the Mortality Table for Prussia, published by R. Boeckh in Hildebrand's Jahrbüchern, Vol. XXV., Nos. 4-5, with valuable explanations. This probability is for all males born (including still-born) in the year 1864 whose ages range from 0 to possibly 1 year; *i.e.*, therefore,

For all at the age 0-1 year	0.27058
" " " " 1-2 years	0.09070
" " " " 2-3 "	0.04942
" " " " 3-4 "	0.02832

According to this ratio of 100,000 persons born (including still-born)—

There died at the age of 0-1 year 27,058

Hence there survived the first year 72,942

Of these the following died at the age of 1-2 years . 6,616

Hence there survived the second year 66,326

Of these the following died at the age of 2-3 years . 3,278

Hence there survived the third year 63,048

Of these the following died at the age of 3-4 years . 1,783

Hence there survived the fourth year 61,265

And so forth, as in column 7 of the table.

Carried through all the ages this calculation shows how many of the 100,000 born die at each age and how many survive it.

If we wish the average expectation of life of the survivors at any age, it is the quotient of the number of years the survivors have to live in succeeding age classes divided by the number of the survivors.

If, therefore, as the table shows, there were, 14.1 among the 100,000 who survived the age of 100,000 years, and for which we could assume by direct observation or estimate that each of these would live $\frac{12.3 + 25.2}{14.1} = \frac{37.5}{14.1}$, or, on the average, 2.67 years; then from the

table we may calculate the average expectation of life from the highest age to the youngest, in the following fashion :

	Number of survivors.	YEARS OF SURVIVAL				Average duration of life in years.
		of the dying in the year.	of the survivors of the year.	In the higher age classes	In total	
Survivors at 100 years . . .	14.1	37.5	2.67
Died 99-100 years old . . .	4.8					
Survivors at 99 years . . .	18.9	1.8	14.1	37.5	53.4	2.86
Died 98-99 years old . . .	11.1					
Survivors at 98 years . . .	30.0	2.5	18.9	53.4	74.8	2.49
Died 97-98 years old . . .	16.6					
Survivors at 97 years . . .	46.6	8.3	30.0	74.8	113.1	2.45
Died 96-97 years old . . .	18.5					
Survivors at 96 years . . .	65.1	8.2	46.6	113.1	167.9	2.58
etc.	col. 7	...	as in col. 7	as in col. 6	col. 6	col. 8

In this way we calculate backward the average expectation of life, or the average duration of life, of each age down to the first. These numbers express the order of mortality for each age up to the highest. It must increase with every fall in the mortality and diminish by every increase in the latter.

If we seek the average expectation of life for the total population, we must multiply the expectation at each age by the number reaching that age, and divide the sum of the results by the number of the population.

That the insight into the expectation of life is very important, and that a comparison of it for different countries or periods may lead to valuable results, needs no demonstration. On the other hand, that the results hold good only for a population of a given place and at a given time—that is, for an aggregate strictly limited in time and space—lies in the nature of the problem.

We can, indeed, unite the deaths and living population for several years and for several districts, but this extension of the field for which our average figures are obtained does not change the essence of the aggregate in any way. The coefficient of mortality corresponds to the average yearly number of deaths of persons of a given age divided by the average number of living persons of the same age.

The calculations require great accuracy and a clear insight into the

problem. They are mathematical in character, or, in other words, simply analytical consequences from given elements. The greatest difficulty lies in the technical statistical processes of collecting these elements.

The older census operations called only for the age of persons, and classed them in a few broad groups. The International Statistical Congress resolved, in 1863, that in enumerations of population and in the classifications of death the year of birth should be given. This has been followed partially. For complete security the more recent enumerations require the exact date of birth.

Greater difficulty is found in the double ascertainment of the age and of the year of birth for the persons dying. It is not easy to obtain the date of birth and death, and one must have also the calculation of the age. If the enumeration is taken at a time other than the close of the year, a corresponding division must be made for the births and deaths. The complete number of still-born, with distinction of sex and time of birth, as in the case of the population at large, and the deaths cannot properly be dispensed with. In particular, care must be taken to avoid the inaccuracies, which, unfortunately, occur so frequently in the returns of age. If all these necessary data are unreliable, or if some of them are missing, the result must be open to much more criticism, since probabilities take the place of facts in the elements of calculation.

The most frequent of these assumptions is the one mentioned above, that of the deaths of persons in a given age, one-half belong to each of the possible years of birth. This assumption is fairly correct for the higher age classes, from 5 or even 3 years upward. It is inapplicable to childhood, however, since the number of those who die in the first month is by far the largest, and this number decreases noticeably from month to month in the first and even in the second year. If, therefore, the days of birth and death, at least for the children, are not noted, or in the first year the months and in the second the quarters distinguished, then nothing remains but to interpolate according to the analogy of other aggregates, whereby the general consideration of the conditions of the particular locality should not be neglected.

From such material for different ages only a limited grouping of ages of 5- or 10-year periods is advisable. If the figures in the first instance give ages only by such periods, the interpolation becomes so difficult that the results cannot be regarded with confidence.

Only very general conclusions can be drawn from the calculation of the relation of all deaths to the living population, the so-called death-rate. As Becker has shown, this general rate may be lower in

one State than in another, which in all age classes has a lower mortality, in case the latter has a larger number of children.

None the less, in default of other material, such estimates as Graunt and Halley applied, even to calculating the population, are indispensable, and must be regarded as proper so long as they are drawn from sufficiently analogous conditions. From fragments even conditional approximations can be gathered. Here may be mentioned the investigation of population in earlier centuries, which has attracted much attention of late. (J. Jastrow: *Die Volkszahl deutscher Städte zur Ende des Mittelalters, etc. Ueberblick über Stand und Mittel der Forschung.* Berlin, 1886.)

The historical development of the calculation of mortality tables has been presented with critical acuteness in G. F. Knapp's second essay: "Zur Theorie des Bevölkerungswechsels, 1874, which covers the field from Graunt to Berg (*Befolknags-Statistik, N. F. II. 3, 1860*); Farr (*English Life Table, 1864*), and Becker. At about the same time with Becker's "Theorie der Sterbetafeln," works on the subject appeared, in 1867, as follows: Wittstein: *Die Mathematische Statistik in ihrer Anwendung*; Lazarus, *Ueber Mortalitätsverhältnisse*, and v. Hermann: *Mortalität und Vitalität in Bayern, Beiträge z. Stat. Bayerns, Heft XVII.*; further, Knapp: *Ueber die Ermittelung der Sterblichkeit, 1868*, and Zeuner, *Abhandlungen aus der mathematischen Statistik, 1869*. In 1874 there followed another group of essays: Knapp's essay, cited above; Becker: *Zur Berechnung von Sterbetafeln, Gutachten für den internationalen Congress, 1874*; Lexis: *Einleitung in die Theorie der Bevölkerungsstatistik, 1875*; Bœckh's *Sterblichkeitstafeln für den Preussischen Staat, 1875*, used above, and Lewin: *Denkschrift für den internationalen statistischen Congress zur Pest., 1876*. Essays of a similar character have been published by Armenante and Perozzo, in the *Annali di Statistica, Vol. I., 1876; Ser. 2, Vol. 12, 1880*. The last named, with a preface by Lexis, appeared in translation in Conrad's *Jahrbüchern, N. F., 1 Bd. p. 162*.

Since the year 1876 R. Bœckh has published, on the basis of the material collected under his supervision in Berlin, yearly mortality tables, which are unexcelled in the character of the material and the care of the processes (*Veröffentlichungen des Statistischen Bureaus der Stadt Berlin, 1878, and following years*). These tables bring all the questions of the equalization of ages, of the still-born, of the date of enumeration, of influx and emigration, and the union with previous years, to a satisfactory issue in the clearest and most unimpeachable manner.

Other works of importance in this field are:

G. Mayr: *Bewegung der Bevölkerung in Königreich Bayern, Beilage zur Stat. v. B. Heft XXXVII., 1878*.

v. Berg: *Eléments démographiques de la Suède*, 1879.

Harald Westergaard: *Die Lehre von der Mortalität und Morbilität*, 1882.

Frhr. v. Fircks: *Absterbeordnung, Mortalität, Lebenserwartung, und durchschnittliche Lebensdauer der Preussischen Bevölkerung*, *Zeitschrift des Preuss. Stat. Bur.*, 1882, p. 137.

A. J. van Pesch: *Table de mortalité pour le Pays Bas, calculée sur les donnés de 1870-1880*, in *Bijdragen van het Statistisch Instituut*, No. 3, 1885.

W. Farr: *Vital Statistics*, edited by N. A. Humphreys, 1885.

The elements for international comparison for the years 1865-1883 were collected by Bodio in the work, *Populazione, Movimento dello stato civile*, 1884.

A special value as evidence of how little statistical probabilities can be generalized beyond the field of observation (§ 111), is to be attached to the data for Iceland, Greenland, and the Faroe islands, as given by Westergaard, to the Deutsche Sterblichkeitstafeln nach den Erfahrungen von 23 Lebensversicherungs-Gesellschaften im Auftrage des Collegiums für Lebensversicherungs-wissenschaft in Berlin, 1883, and to the observations of Jagor, on the Mortality of Natives and Europeans in East India, based on the census of 1881 (*Zeitschrift für Ethnologie*, Jahrgang 13. Heft II. p. 92, 1886).